

HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2022
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT I – PREVIEW TEST 1
Time: 1 hour and 20 minutes

NAME OF STUDENT: _____
SCHOOL CODE: 030014
DATE: _____

This examination paper consists of **7** printed pages and **1** blank page for extra working.

The paper consists of **9** questions.

The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly in the space above.
2. Answer **EACH** question in the **SPACE PROVIDED. SHOW ALL WORKING.**
3. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided.
4. Number your questions **carefully and identically to those on the question paper.**
5. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
 2. Scientific calculator (non-programmable, non-graphical)
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1) Given that a and b are propositions, use the algebra of propositions to simplify fully

$$(\sim p \wedge \sim q) \vee \sim (p \vee \sim q)$$

[4]

Total: 4 marks

2) Prove that for all real numbers x and y , $\frac{x+y}{2} \geq \sqrt{xy}$

[3]

Total: 3 marks

3) Express $\frac{2\sqrt{a} - \sqrt{b}}{2\sqrt{a} + \sqrt{b}}$ as a fraction with a rational denominator, where a and b are positive integers.

[4]

Total: 4 marks

4) Prove by mathematical induction that $8^n + 6$ is divisible by 7 $\forall n \in \mathbb{N}$. [6]

Total: 6 marks

5) The expression $x^3 + px^2 + qx + 12$ is exactly divisible by $x - 1$ and $x + 3$.

(i) Determine the values of p and of q [5]

(ii) Find the third factor of the expression [2]

(ii) Hence, solve $x^3 + px^2 + qx + 12 = 0$. [4]

Total: 11 marks

6) Solve for x

(a) $\log_2(x^2 - x + 2) = 1 + 2\log_2 x$ [5]

(b) $2^{x+3} = 2^{1-x} + 15$. [5]

Total: 10 marks

7) The population, $P(t)$, of larvae found in a pond after t hours is modelled by $P(t) = 250e^{0.02t}$

(a) Determine for the swamp

(i) the initial population of larvae [1]

(ii) the population of larvae after 12 hours [2]

(b) The length of time, in hours, for which the population is first expected to exceed 400. [4]

Total: 7 marks

8) Find the range of values of x for which $\left| \frac{2x-1}{3} \right| \geq 2$. [5]

Total: 5 marks

9) If α, β and γ are the roots of the equation $2x^3 - 11x^2 + 4x + 5 = 0$

(a) Find the values of

(i) $\alpha + \beta + \gamma$

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$

(iii) $\alpha\beta\gamma$

[3]

(b) Hence, or otherwise, find the equation with roots $\alpha + 2, \beta + 2$ and $\gamma + 2$.

[7]

Total: 10 marks

End of Test

EXTRA SPACE

If you use this extra page, you **MUST** write the question number clearly in the box provided.

Question No.