CAPE UNIT 2 (2016 – 2017)

TEST 3 (PREVIEW)

1. Given that $A = \begin{pmatrix} 6 & 8 & -4 \\ -1 & -1 & 1 \\ -8 & -12 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 & 2 \\ -1 & 2 & -1 \\ 2 & 4 & 1 \end{pmatrix}$,

(a) Show that A is a non – singular matrix. [3]

(b) Determine AB. [3]

(c) Hence, deduce A^{-1}, the inverse of A. [2]

(d) Use A^{-1} to solve the system of equations

$$6x + 8y - 4z = -6$$

$$-x - y + z = 2$$

$$-8x - 12y + 6z = 10$$

[3]

(a) $|A| = -2$ (b) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ (c) $\frac{1}{2}\begin{pmatrix} 3 & 0 & 2 \\ -1 & 2 & -1 \\ 2 & 4 & 1 \end{pmatrix}$ (d) $\{x, y, z\} = \{1,0,3\}$

2. Given the following system of equations

$$x + 2y - z = -1$$

$$x - y + 3z = 4$$

$$2x + z = 0$$

(a) Write the system as an augmented matrix. [1]

(b) Use row reduction to solve the system of equations. [5]

(c) Determine the general solution if the last equation of the system was replaced by $2x - 2y + 6x = 8$.

[3]

(b) $\{x, y, z\} = \{-1, 1, 2\}$ (c) $z = t$, $y = \frac{4}{3}t - \frac{5}{3}$, $z = -\frac{5}{3}t + \frac{7}{3}$

3. (a) (i) Solve the differential equation $\frac{dy}{dt} = y \cos t$ to obtain y in terms of t. [4]

    (ii) Given that $y = 50$ when $t = \frac{\pi}{2}$, show that $y = 50e^{-(1-\sin t)}$. [2]

(b) A wave machine at a leisure pool produces waves. The height of the water, y cm, above a fixed point at time t seconds is given by the differential equation

$$\frac{dy}{dt} = y \cos t$$

Given that this height is 50 cm after $\frac{\pi}{2}$ seconds, find, to the nearest centimeter, the height of the water after 6 seconds. [2]

(a) (i) $y = Ae^{\sin t}$ (b) (i) 13.9

4. Find the general solution of the differential equation

$$\frac{dy}{dx} + 2y \tan x = \cos^5 x, \quad 0 < x < \frac{\pi}{2}$$

giving your answer in the form $y = f(x)$. [7]

$$y = \sin x \cos^2 x - \frac{\sin^3 x \cos^2 x}{3} + A \cos^2 x$$

5. Given that for the differential equation

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = 2e^{-t}$$

$x = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$.

Find x in terms of t. [8]

$$x = -\frac{5}{3}e^{2t} + \frac{11}{6}e^{3t} - \frac{1}{6}e^{-t}$$

6. A committee of 7 people, which must contain at least 4 men and at least 1 woman, is to be chosen from 10 men and 9 women.

(i) Find the number of possible committees that can be chosen. [3]

(ii) Find the probability that one particular man, Simon, and one particular woman, Diana, are both on the committee. [2]

(iii) Find the number of possible committees that include either Simon or Diana but not both. [3]

(iv) The committee that is chosen consists of 5 men and 2 women. They queue up randomly in a line for refreshments. Find the probability that the women are not next to each other in the queue. [3]

(i) 28602   (ii) $\frac{1743}{14301}$ (iii) 13860   (iv) $\frac{5}{7}$

7. (i) The digits of the number 3 244 687 can be rearranged to give many different 7 – digit numbers. How many of these 7 – digit numbers are even. [4]

(ii) How many different numbers between 20 000 and 30 000 can be formed using 5 different digits from the digits 1, 2, 3, 6, 7, 8? [2]

(i) 1800   (ii) 120