HARRISON COLLEGE INTERNAL EXAMINATION MARCH **2017**

**CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**
**SCHOOL BASED ASSESSMENT**
**PURE MATHEMATICS**
**UNIT 1 – TEST 1 (PREVIEW)**
**1 hour 20 minutes**

This examination paper consists of **2** printed pages.
This paper consists of **9** questions.
The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES

(i) Write your name clearly on each sheet of paper used
(ii) Answer **ALL** questions
(iii) Number your questions identically as they appear on the question paper and do **NOT write your solutions to different questions** beside each other
(iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

(i) Mathematical formulae
(ii) Scientific calculator (non-programmable, non-graphical)

**1)** Given that $\boldsymbol{p}$ and $\boldsymbol{q}$ are propositions, use the algebra of propositions to simplify fully
$(\boldsymbol{p} \wedge \boldsymbol{q}) \vee (\boldsymbol{p} \wedge \sim \boldsymbol{q})$ [3]

**2)** (a) (i) Express in terms of $n$, $\sum_{\boldsymbol{r=1}}^{\boldsymbol{n+1}} (\boldsymbol{r+2})$. Ans. $\frac{\mathbf{1}}{\mathbf{2}}(n+1)[n+6]$ [3]
(ii) Given that $\sum_{r=1}^{n+1}(r+2) = 7n$, find the possible value(s) of $n$. Ans. $n = 1$ or $n = 6$ [3]

(b) Determine the values of $x$ and $y$ such that $\frac{3\sqrt{2}+5}{2+\sqrt{2}} \equiv x + y\sqrt{2}$ . Ans. 2, $\frac{\mathbf{1}}{\mathbf{2}}$ [3]

**3)** Given that – 1 and 2 are two roots of the equation $x^3 + \boldsymbol{p}x^2 + x + \boldsymbol{q} = 0$.
(i) Find the values of $\boldsymbol{p}$ and $\boldsymbol{q}$. Ans. $p = -4$, $q = 6$ [6]
(ii) Hence or otherwise, find the other root of the equation. Ans. 3 [3]

**4)** Prove by mathematical induction that $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1} \ \forall\, n \in Z^+$. [7]

**5)** Solve for $y \in \boldsymbol{R}$, the equation $6e^{2y} = 7e^{y} + 3$,
giving your answer in terms of logs. Ans. $\ln(\frac{3}{2})$ [5]

6) The population of a town at the beginning of the year 2000 was 2400.

The population increased so that, after a period of $n$ years, the new population was $2400\,(1.06)^{n.}$ Calculate estimates of

(i) the population at the beginning of 2010. Ans. 4298 [3]
(ii) the year in which the population is expected to first reached 7000. Ans. 2018 [4]

**7)** The function $f$ is given by $f: x \to \ln 2x$ and the function $g$ is given by $g: x \to e^{2x}$.

(i) Sketch the graph of $f$, showing clearly any intersection with the axes. [2]
(ii) Find an expression in terms of $x$ for $f^{-1}(x)$. Ans. $\frac{1}{2}e^{x}$ [4]
(iii) State for $f^{-1}(x)$
(a) the domain. Ans. $\boldsymbol{R}$ [1]
(b) the range. Ans. $y > 0$ [1]
(iii) Determine $gf(x)$, simplifying your answer. Ans. $4x^2$ [3]

**8)** Find the range of values of $x \in R$ for which $\frac{x-2}{x-3} \leq 0$, $x \neq 3$. Ans. $2 \leq x < 3$ [4]

**9)** Find the range of values of $x$ such that $|4-3x| \leq x$. Ans. $1 \leq x \leq 2$ [5]

**End of Examination**