1.1

Solving Linear Equations - One Step Equations

Objective: Solve one step linear equations by balancing using inverse operations

Solving linear equations is an important and fundamental skill in algebra. In algebra, we are often presented with a problem where the answer is known, but part of the problem is missing. The missing part of the problem is what we seek to find. An example of such a problem is shown below.

Example 41.

$$4x + 16 = -4$$

Notice the above problem has a missing part, or unknown, that is marked by x. If we are given that the solution to this equation is -5, it could be plugged into the equation, replacing the x with -5. This is shown in Example 2.

Example 42.

$$\begin{aligned} 4(-5) + 16 &= -4 && \text{Multiply } 4(-5) \\ -20 + 16 &= -4 && \text{Add } -20 + 16 \\ -4 &= -4 && \text{True!} \end{aligned}$$

Now the equation comes out to a true statement! Notice also that if another number, for example, 3, was plugged in, we would not get a true statement as seen in Example 3.

Example 43.

$$\begin{aligned} 4(3) + 16 &= -4 && \text{Multiply } 4(3) \\ 12 + 16 &= -4 && \text{Add } 12 + 16 \\ 28 &\neq -4 && \text{False!} \end{aligned}$$

Due to the fact that this is not a true statement, this demonstates that 3 is not the solution. However, depending on the complexity of the problem, this "guess and check" method is not very efficient. Thus, we take a more algebraic approach to solving equations. Here we will focus on what are called "one-step equations" or equations that only require one step to solve. While these equations often seem very fundamental, it is important to master the pattern for solving these problems so we can solve more complex problems.

Addition Problems

To solve equations, the general rule is to do the opposite. For example, consider the following example.

Example 44.

$x+7=-5$	The 7 is added to the x
$\underline{\mathbf{-7 \quad -7}}$	Subtract 7 from both sides to get rid of it
$x=-12$	Our solution!

Then we get our solution, $x=-12$. The same process is used in each of the following examples.

Example 45.

$4+x=8$	$7=x+9$	$5=8+x$
$\underline{\mathbf{-4 \quad -4}}$	$\underline{\mathbf{-9 \quad -9}}$	$\underline{\mathbf{-8 \; -8}}$
$x=4$	$-2=x$	$-3=x$

Table 1. Addition Examples

Subtraction Problems

In a subtraction problem, we get rid of negative numbers by adding them to both sides of the equation. For example, consider the following example.

Example 46.

$x-5=4$	The 5 is negative, or subtracted from x
$\underline{\mathbf{+5 \; +5}}$	Add 5 to both sides
$x=9$	Our Solution!

Then we get our solution $x=9$. The same process is used in each of the following examples. Notice that each time we are getting rid of a negative number by adding.

Example 47.

$$\begin{array}{r} -6+x=-2 \\ \mathbf{+6} \quad \mathbf{+6} \\ \hline x=4 \end{array} \qquad \begin{array}{r} -10=x-7 \\ \mathbf{+7} \quad \mathbf{+7} \\ \hline -3=x \end{array} \qquad \begin{array}{r} 5=-8+x \\ \mathbf{+8} \quad \mathbf{+8} \\ \hline 13=x \end{array}$$

Table 2. Subtraction Examples

Multiplication Problems

With a multiplication problem, we get rid of the number by dividing on both sides. For example consider the following example.

Example 48.

$$\begin{array}{rl} 4x=20 & \text{Variable is multiplied by } 4 \\ \overline{\mathbf{4}} \quad \overline{\mathbf{4}} & \text{Divide both sides by } 4 \\ x=5 & \text{Our solution!} \end{array}$$

Then we get our solution $x=5$

With multiplication problems it is very important that care is taken with signs. If x is multiplied by a negative then we will divide by a negative. This is shown in example 9.

Example 49.

$$\begin{array}{rl} -5x = \ 30 & \text{Variable is multiplied by } -5 \\ \overline{\mathbf{-5}} \quad \overline{\mathbf{-5}} & \text{Divide both sides by } -5 \\ x=-6 & \text{Our Solution!} \end{array}$$

The same process is used in each of the following examples. Notice how negative and positive numbers are handled as each problem is solved.

Example 50.

$\frac{8x}{\mathbf{8}}=\frac{-24}{\mathbf{8}}$	$\frac{-4x}{\mathbf{-4}}=\frac{-20}{\mathbf{-4}}$	$\frac{42}{\mathbf{7}}=\frac{7x}{\mathbf{7}}$
$x=-3$	$x=5$	$6=x$

Table 3. Multiplication Examples

Division Problems:

In division problems, we get rid of the denominator by multiplying on both sides. For example consider our next example.

Example 51.

$$\begin{aligned} \frac{x}{5} &= -3 && \text{Variable is divided by 5} \\ (\mathbf{5})\frac{x}{5} &= -3(\mathbf{5}) && \text{Multiply both sides by 5} \\ x &= -15 && \text{Our Solution!} \end{aligned}$$

Then we get our solution $x = -15$. The same process is used in each of the following examples.

Example 52.

$\frac{x}{-7}=-2$	$\frac{x}{8}=5$	$\frac{x}{-4}=9$
$(\mathbf{-7})\frac{x}{-7}=-2(\mathbf{-7})$	$(\mathbf{8})\frac{x}{8}=5(\mathbf{8})$	$(\mathbf{-4})\frac{x}{-4}=9(\mathbf{-4})$
$x=14$	$x=40$	$x=-36$

Table 4. Division Examples

The process described above is fundamental to solving equations. once this process is mastered, the problems we will see have several more steps. These problems may seem more complex, but the process and patterns used will remain the same.

World View Note: The study of algebra originally was called the “Cossic Art” from the Latin, the study of “things” (which we now call variables).

1.1 Practice - One Step Equations

Solve each equation.

1) $v+9=16$

2) $14=b+3$

3) $x-11=-16$

4) $-14=x-18$

5) $30=a+20$

6) $-1+k=5$

7) $x-7=-26$

8) $-13+p=-19$

9) $13=n-5$

10) $22=16+m$

11) $340=-17x$

12) $4r=-28$

13) $-9=\frac{n}{12}$

14) $\frac{5}{9}=\frac{b}{9}$

15) $20v=-160$

16) $-20x=-80$

17) $340=20n$

18) $\frac{1}{2}=\frac{a}{8}$

19) $16x=320$

20) $\frac{k}{13}=-16$

21) $-16+n=-13$

22) $21=x+5$

23) $p-8=-21$

24) $m-4=-13$

25) $180=12x$

26) $3n=24$

27) $20b=-200$

28) $-17=\frac{x}{12}$

29) $\frac{r}{14}=\frac{5}{14}$

30) $n+8=10$

31) $-7=a+4$

32) $v-16=-30$

33) $10=x-4$

34) $-15=x-16$

35) $13a=-143$

36) $-8k=120$

37) $\frac{p}{20}=-12$

38) $-15=\frac{x}{9}$

39) $9+m=-7$

40) $-19=\frac{n}{20}$

1.2

Linear Equations - Two-Step Equations

Objective: Solve two-step equations by balancing and using inverse opperations.

After mastering the technique for solving equations that are simple one-step equations, we are ready to consider two-step equations. As we solve two-step equations, the important thing to remember is that everything works backwards! When working with one-step equations, we learned that in order to clear a "plus five" in the equation, we would subtract five from both sides. We learned that to clear "divided by seven" we multiply by seven on both sides. The same pattern applies to the order of operations. When solving for our variable x, we use order of operations backwards as well. This means we will add or subtract first, then multiply or divide second (then exponents, and finally any parentheses or grouping symbols, but that's another lesson). So to solve the equation in the first example,

Example 53.

$$4x - 20 = -8$$

We have two numbers on the same side as the x. We need to move the 4 and the 20 to the other side. We know to move the four we need to divide, and to move the twenty we will add twenty to both sides. If order of operations is done backwards, we will add or subtract first. Therefore we will add 20 to both sides first. Once we are done with that, we will divide both sides by 4. The steps are shown below.

$$\begin{array}{rl} 4x - 20 = -8 & \text{Start by focusing on the subtract 20} \\ \underline{+\mathbf{20} + \mathbf{20}} & \text{Add 20 to both sides} \\ 4x \qquad = 12 & \text{Now we focus on the 4 multiplied by } x \\ \overline{\mathbf{4}} \qquad \overline{\mathbf{4}} & \text{Divide both sides by 4} \\ x = 3 & \text{Our Solution!} \end{array}$$

Notice in our next example when we replace the x with 3 we get a true statement.

$$\begin{array}{rl} 4(3) - 20 = -8 & \text{Multiply } 4(3) \\ 12 - 20 = -8 & \text{Subtract } 12 - 20 \\ -8 = -8 & \text{True!} \end{array}$$

The same process is used to solve any two-step equations. Add or subtract first, then multiply or divide. Consider our next example and notice how the same process is applied.

Example 54.

$$\begin{array}{rl} 5x+7= \ 7 & \text{Start by focusing on the plus 7} \\ \underline{-\mathbf{7} \ \ -\mathbf{7}} & \text{Subtract 7 from both sides} \\ 5x \ \ = 0 & \text{Now focus on the multiplication by 5} \\ \overline{\mathbf{5}} \ \ \ \overline{\mathbf{5}} & \text{Divide both sides by 5} \\ x= \ 0 & \text{Our Solution!} \end{array}$$

Notice the seven subtracted out completely! Many students get stuck on this point, do not forget that we have a number for "nothing left" and that number is zero. With this in mind the process is almost identical to our first example.

A common error students make with two-step equations is with negative signs. Remember the sign always stays with the number. Consider the following example.

Example 55.

$$\begin{array}{rl} 4-2x=10 & \text{Start by focusing on the positive 4} \\ \underline{-\mathbf{4} \qquad -\mathbf{4}} & \text{Subtract 4 from both sides} \\ -2x \ =6 & \text{Negative (subtraction) stays on the } 2x \\ \overline{-\mathbf{2}} \ \ \overline{-\mathbf{2}} & \text{Divide by } -2 \\ x=-3 & \text{Our Solution!} \end{array}$$

The same is true even if there is no coefficient in front of the variable. Consider the next example.

Example 56.

$$\begin{array}{rl} 8-x=2 & \text{Start by focusing on the positive 8} \\ \underline{-\mathbf{8} \quad\ -\mathbf{8}} & \text{Subtract 8 from both sides} \\ -x=-6 & \text{Negative (subtraction) stays on the } x \\ -1x=-6 & \text{Remember, no number in front of variable means 1} \end{array}$$

$$\begin{array}{rrl} \overline{\mathbf{-1}} & \overline{\mathbf{-1}} & \text{Divide both sides by } -1 \\ & x=6 & \text{Our Solution!} \end{array}$$

Solving two-step equations is a very important skill to master, as we study algebra. The first step is to add or subtract, the second is to multiply or divide. This pattern is seen in each of the following examples.

Example 57.

$$\begin{array}{r} -3x+7=-8 \\ \underline{\mathbf{-7}\quad\mathbf{-7}} \\ -3x=-15 \\ \overline{\mathbf{-3}}\quad\overline{\mathbf{-3}} \\ x=5 \end{array}$$

$$\begin{array}{r} -2+9x=7 \\ \underline{\mathbf{+2}\qquad\mathbf{+2}} \\ 9x=9 \\ \overline{\mathbf{9}}\quad\overline{\mathbf{9}} \\ x=1 \end{array}$$

$$\begin{array}{r} 8=2x+10 \\ \underline{\mathbf{-10}\qquad\mathbf{-10}} \\ -2=2x \\ \overline{\mathbf{2}}\quad\overline{\mathbf{2}} \\ -1=x \end{array}$$

$$\begin{array}{r} 7-5x=17 \\ \underline{\mathbf{-7}\qquad\mathbf{-7}} \\ -5x=10 \\ \overline{\mathbf{-5}}\quad\overline{\mathbf{-5}} \\ x=-2 \end{array}$$

$$\begin{array}{r} -5-3x=-5 \\ \underline{\mathbf{+5}\qquad\mathbf{+5}} \\ -3x=0 \\ \overline{\mathbf{-3}}\quad\overline{\mathbf{-3}} \\ x=0 \end{array}$$

$$\begin{array}{r} -3=\frac{x}{5}-4 \\ \underline{\mathbf{+4}\qquad\mathbf{+4}} \\ (\mathbf{5})(1)=\frac{x}{5}(\mathbf{5}) \\ 5=x \end{array}$$

Table 5. Two-Step Equation Examples

As problems in algebra become more complex the process covered here will remain the same. In fact, as we solve problems like those in the next example, each one of them will have several steps to solve, but the last two steps are a two-step equation like we are solving here. This is why it is very important to master two-step equations now!

Example 58.

$$3x^2+4-x+6 \qquad \frac{1}{x-8}+\frac{1}{x}=\frac{1}{3} \qquad \sqrt{5x-5}+1=x \qquad \log_5(2x-4)=1$$

World View Note: Persian mathematician Omar Khayyam would solve algebraic problems geometrically by intersecting graphs rather than solving them algebraically.

1.2 Practice - Two-Step Problems

Solve each equation.

1) $5+\frac{n}{4}=4$

2) $-2=-2m+12$

3) $102=-7r+4$

4) $27=21-3x$

5) $-8n+3=-77$

6) $-4-b=8$

7) $0=-6v$

8) $-2+\frac{x}{2}=4$

9) $-8=\frac{x}{5}-6$

10) $-5=\frac{a}{4}-1$

11) $0=-7+\frac{k}{2}$

12) $-6=15+3p$

13) $-12+3x=0$

14) $-5m+2=27$

15) $24=2n-8$

16) $-37=8+3x$

17) $2=-12+2r$

18) $-8+\frac{n}{12}=-7$

19) $\frac{b}{3}+7=10$

20) $\frac{x}{1}-8=-8$

21) $152=8n+64$

22) $-11=-8+\frac{v}{2}$

23) $-16=8a+64$

24) $-2x-3=-29$

25) $56+8k=64$

26) $-4-3n=-16$

27) $-2x+4=22$

28) $67=5m-8$

29) $-20=4p+4$

30) $9=8+\frac{x}{6}$

31) $-5=3+\frac{n}{2}$

32) $\frac{m}{4}-1=-2$

33) $\frac{r}{8}-6=-5$

34) $-80=4x-28$

35) $-40=4n-32$

36) $33=3b+3$

37) $87=3-7v$

38) $3x-3=-3$

39) $-x+1=-11$

40) $4+\frac{a}{3}=1$