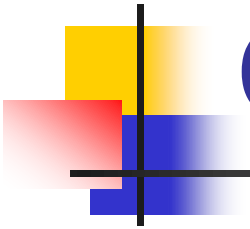
First Form Mathematics

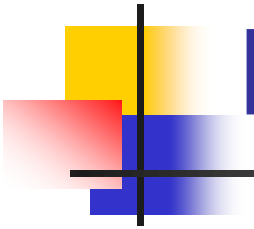
LESSON 1 on

Number Bases



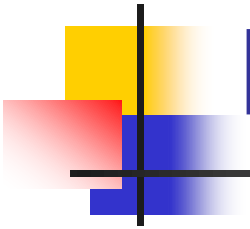
Objective

- In this lesson you'll learn about different Number Bases, specifically about those used by the computer
- Those include:
  - Base Two – binary
  - Base Eight – octal
  - Base Sixteen – hexadecimal



Base Ten

- First let's talk about base ten, the decimal number system which humans use and you have been working with for years.
- It's called base ten because…?



Base Ten

- If you said, "because it has ten counting digits, 0,1,2,3,4,5,6,7,8, and 9", you are right!
- To count in base ten, you go from 0 to 9, then do combinations of two digits starting with 10 all the way to 99

Base Ten

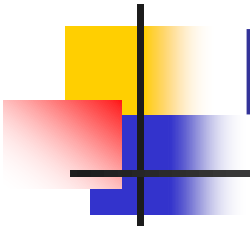
- After 99 comes three-digit combinations from 100 – 999, etc.
- This combination system is true for any base you use.
- The only difference is how many digits you have before you go to the next combination

Base Two

- To count in base two, which only has 0 and 1 as counting digits, you count 0,1, then switch to two digit combinations, 10,11, then to three digit combos, 100, 101,110,111, then four digit, 1000, _____,_____, …, 1111

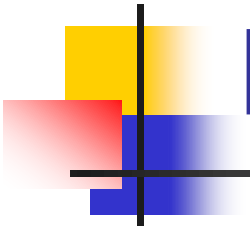
Base Three

- To count in base three, which has 0, 1, and 2 as counting digits, you count 0,1,2, then switch to two digit combinations, 10,11, 12, 20, 21, 22, then to three digit combos, 100, 101,102, 110,111, 112, etc…



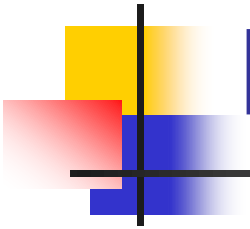
Base Eight

- Jumping to base eight (often called ***octal***)… what are the counting digits?
- Can you count correctly using single digits, two-digit combinations, and then three-digit combos?



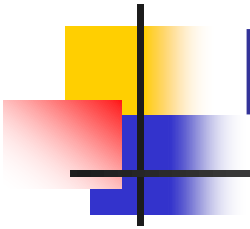
Base Eight

- Here is the base eight counting sequence
  - 0,1,2,3,4,5,6,7,10,11,12,13,…77
  - 100,101,102,103,104,105,106,107
  - 110,111, etc.



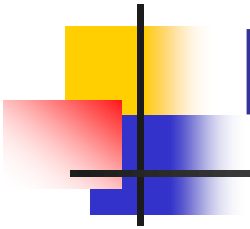
Base Sixteen

- Now for one that's a bit strange.
- Base Sixteen, also known as *hexadecimal,* was especially created by computer scientists to help simplify low-level programming, like machine language and assembly language.



Base Sixteen

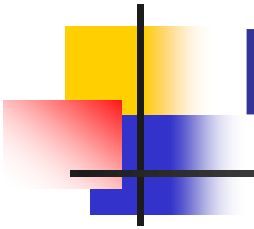
- To count in base sixteen, you need 16 counting digits.
- To get sixteen counting digits, you use 0-9, but still need six more…so it was decided to use A,B,C,D,E, and F.



Base Sixteen

- The symbol **A** represents the value 10, **B** is 11, **C** is 12, **D** is 13, **E** is 14, and **F** is 15.
- Here's the single digit sequence for base sixteen:

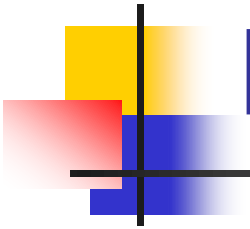
0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F



Base Sixteen

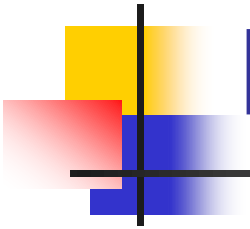
- Then the two-digit combos:

10,11,12,…19,1A,1B,1C,1D,1E,1F,
20,21,22,…2D,2E,2F,30,31,…FF



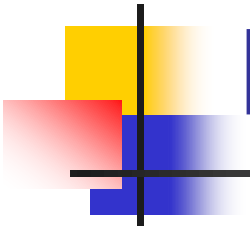
Base conversion

- To convert from base ten to another base, such as base two, eight, or sixteen, is an important skill for computer scientists and programmers.
- The next section shows how to do this.



Base Ten to Base Two

- Let's take the value 27 and convert it into base 2.
- Here's the process:
  - Divide 27 by 2
  - The answer is 13, remainder 1
  - Divide 13 by 2
  - Answer is 6, remainder 1

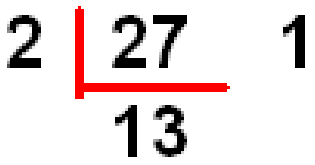


Base Ten to Base Two

- Continue until the answer is 1.
  - 6 divided by 2 = 3, remainder 0
  - 3 divided by 2 = 1, remainder 1
- Now take the last answer, 1, and all of the remainders in reverse order, and put them together…11011
- 27 base 10 = 11011 base two

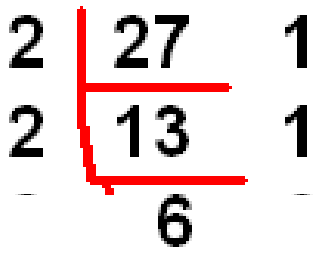
Base Ten to Base Two

- Here's an easy way to do it on paper



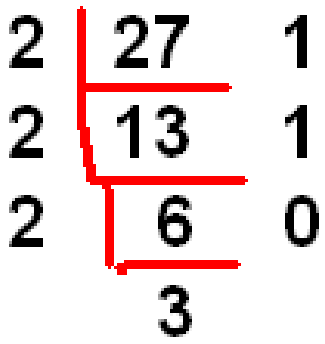
- 27 divided by 2 = 13, R 1

Base Ten to Base Two



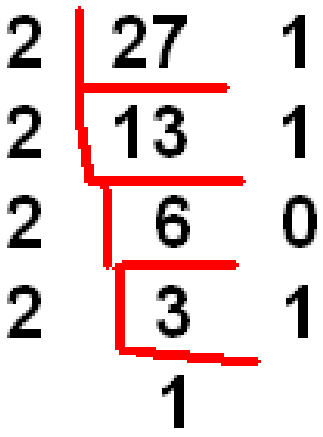
- 13 / 2 = 6, R 1

Base Ten to Base Two



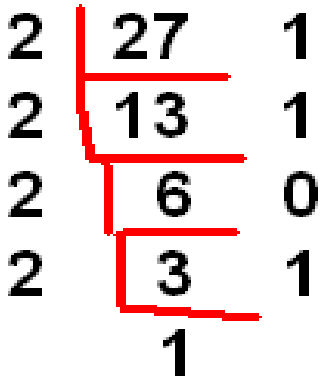
- 6 / 2 = 3, R 0

Base Ten to Base Two



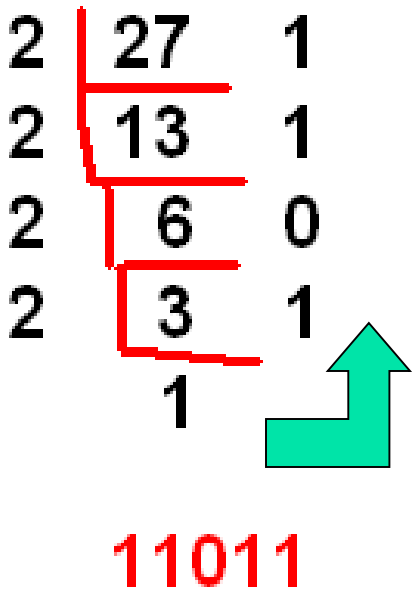
- 3 / 2
  = 1, R 1

Base Ten to Base Two



- Stop, and write the answer

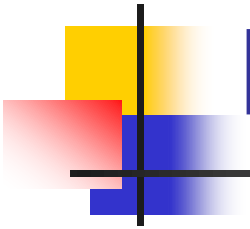
Base Ten to Base Two



Exercises

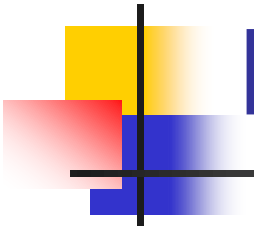
- Now try a few yourself (see last slide for answers):

1. $16_{10} =$ _____$_2$
2. $47_{10} =$ _____$_2$
3. $145_{10} =$ _____$_2$
4. $31_{10} =$ _____$_2$
5. $32_{10} =$ _____$_2$



Base Ten to Base Eight

- Let's again take the value 27 and convert it into base 8.
- Same process:
  - Divide 27 by 8
  - The answer is 3, remainder 3
  - Stop! You can't divide anymore because the answer is less than 8



Base Ten to Base Eight

- The last answer was 3, and the only remainder was 3, so the base eight value is 33, base 8.

Base Ten to Base Eight

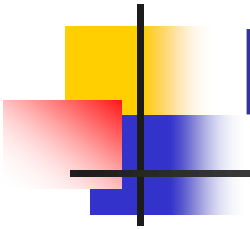
- Use the same method on paper

- 27 divided by 8 = 3, R 3
- 27, base 10 = 33, base 8

Exercises

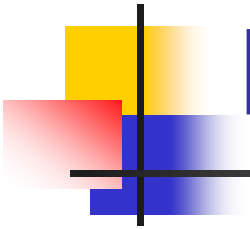
- Now try the same values for base eight.

6. $16_{10} =$ _____$_8$
7. $47_{10} =$ _____$_8$
8. $145_{10} =$ _____$_8$
9. $31_{10} =$ _____$_8$
10. $32_{10} =$ _____$_8$



Base Ten to Base Sixteen

- Finally we'll convert 27 into base 16.
  - Divide 27 by 16
  - The answer is 1, remainder 11
  - Stop! You can't divide anymore because the answer is less than 16

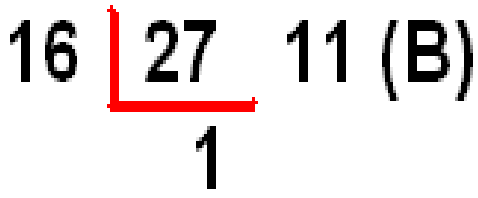


Base Ten to Base Sixteen

- The last answer was 1, and the only remainder was 11, which in base 16 is the letter B, so the base sixteen value is 1B, base 16.

Base Ten to Base Sixteen

- Again, the same method on paper

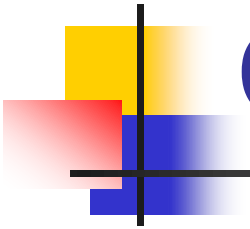


- 27 divided by 16 = 1, R 11 or B
- 27, base 10 = 1B, base 16

Exercises

- And now try base sixteen!

11. $16_{10} =$ _____$_{16}$
12. $47_{10} =$ _____$_{16}$
13. $145_{10} =$ _____$_{16}$
14. $31_{10} =$ _____$_{16}$
15. $32_{10} =$ _____$_{16}$



Conclusion

- Now you should know
  - how to count in different bases
  - how to convert from
    - Base ten to base 2
    - Base ten to base 8
    - Base ten to base 16

Here are the answers to the exercises, in jumbled order

10 1F 20 20 2F 37 40
57 91 221 10000 11111
101111 100000 10010001