SBA 2016

i) a) $(x+2)^2 + (y-3)^2 = 5$

1 Correct values for h and k

1 correct value for r

b) $x^2 + y^2 - 8x - 6y = -16$

$(x-4)^2 + (y-3)^2 = -16 + 16 + 9$ 1 writing in the form $(x-h)^2 + (y-k)^2 = r^2$

$(x-4)^2 + (y-3)^2 = 9$

$C(4,3) \qquad r = 3$ 1 coordinates of centre

grad of $OB = \dfrac{3-0}{4-11} = -\dfrac{3}{7}$ 1 calculating gradient of OB

$y = mx + c$

$0 = -\dfrac{3}{7}(11) + c$

$\dfrac{33}{7} = c$ 1 correct value of c

$y = -\dfrac{3}{7}x + \dfrac{33}{7}$ 1 correct equation

ii) $|OB| = \sqrt{(4-11)^2 + (3-0)^2}$ 1 correct use of length formula

$= \sqrt{58}$ 1 length of OB

$|OA| = 3$ 1 using his radius

$AB^2 = (\sqrt{58})^2 - 3^2$ 1 correct use of Pythagoras' Theorem

$= 49$

$AB = 7$ 1 C.A.O

Question 2

2) a) $\sin^4\theta - \cos^4\theta + 1$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta) + 1$ — 1 Difference of 2 squares

$= \sin^2\theta - \cos^2\theta + 1$ — 1 use of $\sin^2\theta + \cos^2\theta = 1$

$= \sin^2\theta - (1 - \sin^2\theta) + 1$ — 1 use of $\cos^2\theta = 1 - \sin^2\theta$

$= 2\sin^2\theta$ — 1 simplifying

b) $\sec\theta + 5\tan\theta = 3\cos\theta$

$\frac{1}{\cos\theta} + \frac{5\sin\theta}{\cos\theta} = 3\cos\theta$ — 1 Rewriting $\sec\theta$ and $\tan\theta$

$1 + 5\sin\theta = 3\cos^2\theta$

$1 + 5\sin\theta = 3(1 - \sin^2\theta)$

$1 + 5\sin\theta = 3 - 3\sin^2\theta$ — 1 use of $1 - \sin^2\theta = \cos^2\theta$

$3\sin^2\theta + 5\sin\theta - 2 = 0$

$(3\sin\theta - 1)(\sin\theta + 2) = 0$ — 1 mark for each solution of quadratic equation in terms of $\sin\theta$

$\sin\theta = \frac{1}{3}$ $\quad$ $\sin\theta = -2$ INVALID

$R.A = \sin^{-1}\left(\frac{1}{3}\right)$

$= 0.34^c$ — 1 Reference angle

sine is positive in I and II

I: $\theta = 0.34^c$ — 1 angle in I

II: $\theta = \pi - 0.34 = 2.80^c$ — 1 angle in II

Question 2 cont'd

c i) $\sin 2A = 2\sin A\cos A$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$ ✓ use of $\cos A = \frac{4}{5}$

$= \frac{24}{25}$ ✓ c.A.O

b) $\cos 2A = 2\cos^2 A - 1$

$= 2\left(\frac{4}{5}\right)^2 - 1$ ✓ sub. $\cos A = \frac{4}{5}$ in appropriate double angle formula

$= \frac{7}{25}$ ✓ C.A.O

c) $\cos 3A = \cos(2A + A)$ ✓ correct use of compound angle formulae

$= \cos 2A\cos A - \sin 2A\sin A$

$= \left(\frac{7}{25}\right)\left(\frac{4}{5}\right) - \left(\frac{24}{25}\right)\left(\frac{3}{5}\right)$ ✓ use of his values for $\sin 2A$ and $\cos 2A$

$= -\frac{44}{125}$ ✓ his correct answer

ii) Since $\sin 2A$ and $\cos 2A$ are both positive $2A$ is in I ✓ correct Quadrant ✓ correct reason

d) i) $R = \sqrt{5^2 + 12^2} = 13$ ✓ value of R

$\alpha = \tan^{-1}\left(\frac{12}{5}\right) = 1.176^c$ ✓ value of α (in radians)

$13\sin(\theta + 1.176^c)$ ✓ sub. values of R and α

ii) No, since maximum value is 13 ✓ correct answer ✓ correct reason

Question 2

iii) $13 \sin(\theta + 1.176^c) = 2$

$\sin(\theta + 1.176^c) = \frac{2}{13}$

$R.A = \sin^{-1}\left(\frac{2}{13}\right) = 0.154^c$ — 1 correct reference angle

sine is positive in I and II

I: $\theta + 1.176^c = 0.154^c \rightarrow$ INVALID — 1 ignoring I value

II: $\theta + 1.176^c = \pi - 0.154^c$

$= 3$

$\theta = 1.824^c$ — 1 value in II

I (second revolution)

$\theta + 1.176 = 2\pi + 0.154^c$ — 1 use of 2nd revolution

$= 6.44^c$

$\theta = 5.26$ — 1 value for 2nd revolution

3) i) $$\frac{\begin{pmatrix}-1\\2\\3\end{pmatrix}\cdot\begin{pmatrix}4\\2\\-3\end{pmatrix}}{\sqrt{(-1)^2+2^2+3^2}\sqrt{4^2+2^2+3^2}} = \cos\theta$$

1 correct vectors used

1 value of scalar product

$\frac{-9}{\sqrt{14}\sqrt{29}} = \cos\theta$ — 1 values for lengths

$\theta = \cos^{-1}\left(\frac{-9}{\sqrt{14}\sqrt{29}}\right)$

$= 117^\circ$ — 1 C.A.O

ii) $\overrightarrow{AB} = 5i - 6k$ 1 determining $\overrightarrow{AB}$

$|AB| = \sqrt{5^2 + 6^2}$ 1 length of $\overrightarrow{AB}$

$= \sqrt{61}$

unit vector $= \frac{1}{\sqrt{61}}\begin{pmatrix} 5 \\ 0 \\ -6 \end{pmatrix}$ 1 correct form for unit vector

iii) $r \cdot n = a \cdot n$

$r \cdot \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix}$ 1 correct vector for a

1 correct vector for n

$r \cdot \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix} = 17$ 1 corrects scalar product

4) a) $x = \frac{1}{t}$ $y = \frac{2 - t^2}{t}$

$t = \frac{1}{x}$ 1 correct expression for t

$y = \left(2 - \frac{1}{x^2}\right) \times x$ 1 sub $t = \frac{1}{x}$ into y

$= \left(\frac{2x^2 - 1}{x^2}\right) \times x$ 1 simplifying numerator

$= \frac{2x^2 - 1}{x}$ 1 C·A·O

4) b) $x = \cos t$

$x^2 = \cos^2 t$ 1 squaring $x = \cos t$

$y = 3 + 2\cos 2t$

$= 3 + 2(2\cos^2 t - 1)$ 1 use of $\cos 2t = 2\cos^2 t - 1$

$= 3 + 4\cos^2 t - 2$

$= 3 + 4x^2 - 2$ 1 sub $x^2 = \cos^2 t$

$= 1 + 4x^2$ 1 simplifying