

FORM TP 2022331



TEST CODE 02234032

MAY/JUNE 2022

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 032

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of ONE question.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – Revised 2012

Mathematical instruments

Silent, non-programmable electronic calculator

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SECTION A

Module 1

Answer this question.

1. (a) (i) On an Argand diagram, sketch the loci of the points that move in the complex plane such that $|z - 3 - 2i| = 5$ and $|z - 7| = |z + 1|$.

[5 marks]

- (ii) Determine the point(s) of intersection of the two loci described in (a) (i).

[5 marks]

- (b) Use the trapezium rule with 4 intervals to approximate the value of $\int_3^7 \frac{\ln x}{x} dx$.

[4 marks]

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- (c) Determine $\int \frac{x^2}{\sqrt{5-x^2}} dx$, using the substitution $x = \sqrt{5} \sin \theta$.

[6 marks]

Total 20 marks

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SECTION B

Module 2

Answer this question.

2. (a) (i) Use the Intermediate Value Theorem to prove that $f(x) = e^x - 4x - 6$ has a root in the interval $[2, 3]$.

[3 marks]

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- (ii) Hence, or otherwise, use FOUR iterations of the Newton–Raphson method with the initial approximation of 2.5 to find the root of f in the interval $[2, 3]$.

[6 marks]

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- (b) Use the iteration $x_{n+1} = \frac{8 - \ln(4 + x_n)}{3}$ with initial approximation $x_1 = 1.5$ to estimate the root of the equation $\ln(4 + x) = 8 - 3x$, correct to two decimal places.

[5 marks]

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- (c) A sequence is given as $\mu_{n+1} = 3\mu_n + 4$ with $\mu_1 = 1$ for $n \in \mathbb{N}$. Prove, by mathematical induction, that $\mu_n = 3^n - 2$.

[6 marks]

Total 20 marks

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SECTION C

Module 3

Answer this question.

3. (a) A basketball player hits 70% of her free throws. For every two free throws taken, the four possible simple events and three of their associated probabilities are given in the table below.

Outcome of First Free Throw	Outcome of Second Free Throw	Probability
Hit	Hit	0.49
Hit	Miss	
Miss	Hit	0.21
Miss	Miss	0.09

- (i) Determine the probability that the player will hit the first free throw and miss the second one.

[2 marks]

- (ii) Calculate the probability that the player will hit **at least one** of the two free throws taken.

[3 marks]

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- (b) The letters from the word **RELATION** are to be selected and arranged in a row.
- (i) In how many ways can three of the letters be selected and arranged?

[3 marks]

- (ii) How many of the arrangements of three letters from the word will contain exactly ONE vowel?

[5 marks]

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- (c) A differential equation is given as $\sin x \frac{dy}{dx} + y \cos x = x^2$.

Determine the solution of the differential equation given that $y = \frac{1}{3}$ when $x = \frac{\pi}{2}$.

[7 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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