

HARRISON COLLEGE INTERNAL EXAMINATION 2021
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT I – TEST 1
Time: 1 hour and 20 minutes

NAME OF STUDENT: _____
SCHOOL CODE: 030014
DATE: MAY 2021

This examination paper consists of **8** printed pages and **1** blank page for extra working.

The paper consists of **9** questions.

The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly in the space above.
2. Answer **ALL** questions in the **SPACES PROVIDED**.
3. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided.
4. Number your questions **carefully and identically to those on the question paper**.
5. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
 2. Scientific calculator (non-programmable, non-graphical)
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- 1) Given that a and b are propositions, use the algebra of propositions to simplify fully
 $(\sim p \wedge \sim q) \vee \sim(p \vee \sim q)$ [4]

SOLUTION

$\sim p$

- 2) Prove that for all real numbers x and y , $\frac{x^2 + y^2}{2} \geq xy$ [3]

- 3) Express $\frac{2\sqrt{a} - \sqrt{b}}{2\sqrt{a} + \sqrt{b}}$ as a fraction with a rational denominator, where a and b are positive integers. [4]

SOLUTION

$$\frac{4a - 4\sqrt{ab} + b}{4a - b}$$

- 4) Prove by mathematical induction that $8^n + 6$ is divisible by 7. [6]

- 5) The cubic $P(x) \equiv mx^3 + 8x^2 + nx + 6$ is exactly divisible by $x^2 - 2x - 3$.

(i) Determine the values of m and of n . [7]

(ii) Hence, solve $P(x) = 0$. [4]

SOLUTION

(i) $m = -5, n = 19$

(ii) $x = 3, x = -1, x = -\frac{2}{5}$

- 6) Solve for x

(a) $\log_2(2x - 1) = \log_4 x$. [5]

(b) $9^x - 7(3^x) = 8$. [5]

SOLUTION

(a) $x = \frac{1}{4}$ (*not valid*), $x = 1$

(b) $x = \frac{\ln 8}{\ln 3}$, NO SOLN

7) The population, $P(t)$, of larvae found in a pond after t hours is modelled by $P(t) = 250e^{0.02t}$

(a) Determine for the swamp

(i) the initial population of larvae [1]

(ii) the population of larvae after 12 hours [2]

(b) The length of time, in hours, for which the population is first expected to exceed 400. [4]

SOLUTION

(a) (i) 250 (ii) 318 (b) $t = 24$ hours

8) Find the range of values of x for which $|2x - 3| - \frac{1}{2}x < 0$. [5]

SOLUTION

$$\frac{6}{5} < x < 2$$

9) If α, β and γ are the roots of the equation $2x^3 - 11x^2 + 4x + 5 = 0$

(a) Find the values of

(i) $\alpha + \beta + \gamma$

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$

(iii) $\alpha\beta\gamma$ [3]

(b) Hence, or otherwise, find the equation with roots $\alpha + 2, \beta + 2$ and $\gamma + 2$. [7]

SOLUTION

(a) (i) $\frac{11}{2}$ (ii) 2 (iii) $-\frac{5}{2}$

(b) $2x^3 - 23x^2 + 72x - 63 = 0$