HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2022
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT I – TEST 1
Time: 1 hour and 20 minutes

NAME OF STUDENT: _____
SCHOOL CODE: 030014
DATE: _____

This examination paper consists of 9 printed pages and **1** blank page for extra working.

The paper consists of **9** questions.

The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly in the space above.
2. Answer **EACH** question in the **SPACE PROVIDED**. **SHOW ALL WORKING.**
3. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided.
4. Number your questions **carefully and identically to those on the question paper**.
5. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Scientific calculator (non-programmable, non-graphical)

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1) Given that $\boldsymbol{p}$ and $\boldsymbol{q}$ are propositions, use the algebra of propositions to simplify fully

$\boldsymbol{\sim (p \vee q) \vee (\sim p \wedge q)}$ [4]

Total: 4 marks

2) Prove that for all $x \in \boldsymbol{R}, y \in \boldsymbol{R}; x \geq 0, y \geq 0; x + y \geq -2\sqrt{\boldsymbol{xy}}$ [3]

Total: 3 marks

3) Simplify <u>FULLY</u> $\frac{\sqrt{p}-\sqrt{2}}{\sqrt{p}+\sqrt{2}}-\frac{\sqrt{p}+\sqrt{2}}{\sqrt{p}-\sqrt{2}}$. [4]

Total: 4 marks

4) Prove by mathematical induction that $\mathbf{9^{2n} - 1}$ is divisible by 8 $\forall \boldsymbol{n} \in \boldsymbol{N}$. [6]

Total: 6 marks

5) The expression $2x^3 + ax^2 + bx + 1$ is exactly divisible by $2x - 1$ and $x + 1$.

(i) Determine the values of a and of b [5]

(ii) Find the third factor of the expression [2]

(ii) Hence, solve $2x^3 + ax^2 + bx + 1 = 0$. [4]

Total: 11 marks

6) Solve for x

(a) $log_2(2x^2 + 3x + 5) = 3 + log_2(x + 1)$ [5]

(b) $2^{2x+1} - 15(2^x) = 8$ [5]

Total: 10 marks

7) The number of mosquito larvae found in a pond initially was 4 000.

The number of larvae after *t* days, *N*(t), was found to be directly proportional to $\left(\frac{3}{2}\right)^{t}$.

Calculate

(i) the number of larvae after 3 days [3]

(ii) the number of days for which the population is first expected to exceed 50 000. [4]

Total: 7 marks

8) Find the range of values of x for which $\left|\mathbf{1} - \frac{\boldsymbol{x}}{\mathbf{3}}\right| < \mathbf{3}$. [5]

Total: 5 marks

9) If $\boldsymbol{\alpha, \beta}$ **and** $\boldsymbol{\gamma}$ are the roots of the equation $3x^3 - 4x^2 - 5x + 2 = 0$

(a) Find the values of

(i) $\boldsymbol{\alpha + \beta + \gamma}$

(ii) $\boldsymbol{\alpha\beta + \alpha\gamma} + \boldsymbol{\beta\gamma}$

(iii) $\boldsymbol{\alpha\beta\gamma}$ [3]

(b) Hence, or otherwise, find the equation with roots $\boldsymbol{\alpha - 1, \beta - 1}$ **and** $\boldsymbol{\gamma - 1}$. [7]

Total: 10 marks

End of Examination

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No.