CAPE Unit 2 Test 3 Preview 2021

1. Roger and Andy play a tennis match in which the first person to win two sets wins the match. The probability that Roger wins the first set is 0.7. For sets after the first, the probability that Roger wins the set is 0.6 if he won the previous set and is 0.35 if he lost the previous set. No set is drawn.
   (i) Find the probability that there is a winner of the match after exactly two sets.
   (ii) Find the probability that Andy wins the match given that there is a winner of the match after exactly two sets.

(i) 0.615 (ii) $\frac{13}{41}$

2. A group of 5 performers needs to be selected from 12 applicants consisting of 7 females and 5 males. Anna would like the group of 5 performers to contain more males than females.
   (a) Find the number of different selections of 5 performers with more males than females.
   (b) Two of the applicants are Mr and Mrs Fields. Find the number of different selections that include Mr and Mrs Fields and also fulfils Anna's requirement.

(a) 246 (b) 40

3. Find how many different numbers can be made from some or all of the digits of the number 1 345 789 if
   (i) all seven digits are used, the even digits are all together and no digits are repeated,
   (ii) the numbers made are odd numbers between 3000 and 5000, and no digits are repeated,
   (iii) the numbers made are multiples of 5 which are less than 100, and digits can be repeated.

(i) 1440 (ii) 180 (ii) 8

4. A and B are two matrices given below.

$$A = \begin{pmatrix} 2 & x & -1 \\ 3 & 0 & 2 \\ 2 & 1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 4 \\ 2 & 1 & 2 \end{pmatrix}$$

   (i) Determine the range of values of x for which A^{-1} exists. [4]
   (ii) Given that $\det(AB) = -10$, show that $x = 2$. [4]
   (iii) Hence, obtain A^{-1}. [4]

(i) $x \neq \frac{7}{4}$ (iii) $\begin{pmatrix} -2 & -1 & 4 \\ 4 & 2 & -7 \\ 3 & 2 & -6 \end{pmatrix}$

5. Solve, using row reduction, the system of equations

$$x - y + z = 7$$
$$x + 2y - z = -1$$
$$4y - z = -9$$

$x = 4, y = -2, z = 1$

6. Solve the differential equation $x\frac{dy}{dx} - 4y = x^5 e^{2x}$ for y in terms of x, given that $y = 0$ when $x = 1$.

$$y = \frac{1}{2}x^4(e^{2x} - e^2)$$

7. Find the solution of the differential equation $y'' + 2y' + 5y = e^x$ for which $y = y' = 0$ when $x = 0$.

$$y = -\frac{1}{8}e^{-x}(\sin 2x + \cos 2x) + \frac{1}{8}e^x$$