CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 2 – TEST 3

Time: 1 Hour & 20 minutes

This examination paper consists of 2 printed pages.

The paper consists of 4 questions.

The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Number your questions carefully and do **NOT** write your solutions to different questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Electronic calculator (non – programmable, non – graphical)

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1. Candidates applying for jobs in a large company take a test, as a result of which they are either accepted, rejected or retested, with probabilities 0.25, 0.45 and 0.3 respectively. When a candidate is retested for the first time, the three possible outcomes and their probabilities remain the same as for the original test. When a candidate is retested for the second time there are just two possible outcomes, accepted or rejected with probabilities 0.35 and 0.65 respectively.
   (i) Draw a probability tree diagram to illustrate the outcomes. [3]
   (ii) Find the probability that a randomly selected candidate is accepted. [2]
   (iii) Find the probability that a randomly selected candidate is retested at least once, given that this candidate is accepted. [3]

TOTAL 8 Marks

2. A car park has spaces for 15 cars, arranged in a line. On one day there are 6 cars, of different makes in randomly chosen positions and 9 empty spaces.

   (i) Find the number of possible arrangements of the 6 cars. [2]

   (ii) Find the probability that the 6 cars are not all next to each other. [5]

   On another day, 9 cars of different makes are parked in the car park. 4 of these cars are red, 3 are white and 2 are black. Danielle selects 3 of these cars.

   (iii) Find the number of selections Danielle can make that include cars of at least 2 different colours. [5]

TOTAL 12 Marks

3. (a) Use an integrating factor to find the solution of the differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = 2x^3e^{2x}$$

   given that $y = e^4$ when $x = 2$. Give your answer in the form $y = f(x)$. [9]

   (b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 3x^2 + 5$$

[8]

TOTAL 17 Marks

4. A small company manufactures x shirts, y pants and z skirts each month. During the last three months the costs of the raw materials for making the clothes has increased, resulting in various monthly costs as shown in the table below.

Month	Unit Cost ($)			Total Cost ($)
	Shirt	Pants	Skirt	
January	10	15	5	550
February	8	16	8	640
March	10	10	10	700

   (i) Use the information in the table above to form a system of linear equations in x, y and z. [3]

   (ii) Express the system in the form $AX = B$, where A is a 3×3 matrix and X and B are 3×1 matrices. [3]

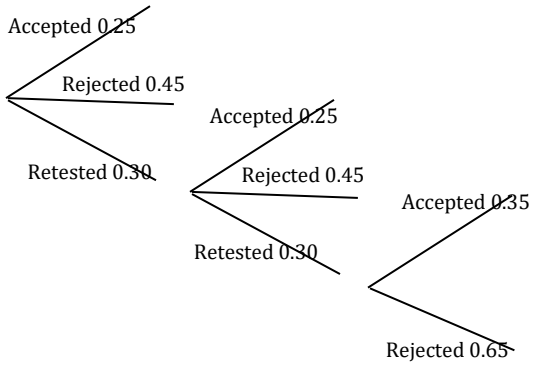
   (iii) Calculate, $|A|$, the determinant of the matrix A. [5]

   (iv) Hence, calculate the inverse of the matrix A. [7]

   (v) Hence, solve the equations for x, y and z. [5]

TOTAL 23 Marks

END OF TEST

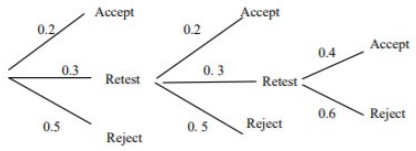
1. (i)		1 mark for each correct branch
(ii)	$P(\text{accepted}) = 0.25 + (0.3 \times .25) + (0.3 \times 0.3 \times 0.35)$ $= \frac{713}{2000} = 0.3565$	$1 - 0.25 + (0.3 \times 0.25) + (0.3 \times 0.3 \times 0.35)$ $1 - \frac{713}{2000}$ or 0.3565 or 0.357
(iii)	$P(\text{Retested at least once}\|\text{Accepted})$ $= \frac{(0.3 \times 0.25) + (0.3 \times 0.3 \times 0.35)}{0.3565}$ $= \frac{213}{713}$ $= 0.300$	1 - Use of $P(A\|B) = \frac{P(A \cap B)}{P(A)}$ 1 – Evaluation of $P(\text{Retested at least once}\|\text{Accepted})$ 1 – His correct answer using his value for $P(\text{Accepted})$
	Total	**7 Marks**
2. (i)	${}^{15}P_6 = 3,603,600$	1 - ${}^{15}P_6$ 1 – C.A.O
(ii)	Number of ways where all 6 cars are together $10 \times 6! = 7,200$ Number of ways where all 6 cars are NOT together $3,603,600 - 7,200 = 3,596,400$ Probability $= \frac{3,596,400}{3,603,600} = \frac{999}{1001} \approx 0.998$	1 – determining the number of ways where all 6 cars are together 1 – determining number of ways where all 6 cars are NOT together 1 – determining probability where all cars are NOT together 1 – subtracting probability from 1 1 – C.A.O
(iii)	At least 2 different colours 1R, 1W, 1B: ${}^4C_1 \times {}^3C_1 \times {}^2C_1 = 24$ 0R, 2W, 1B: ${}^4C_0 \times {}^3C_2 \times {}^2C_1 = 6$	1 – determining 1 different colours 1 – determining combinations involving 2 whites and 1 other colour

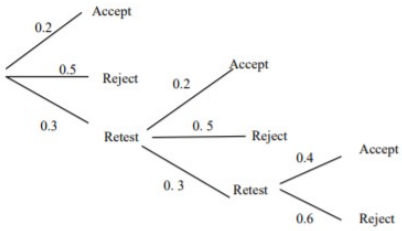
	0R, 1W, 2B: ${}^4C_0 \times {}^3C_1 \times {}^2C_2 = 3$ 1R, 2W, 0B: ${}^4C_1 \times {}^3C_2 \times {}^2C_0 = 12$ 1R, 0W, 2B: ${}^4C_1 \times {}^3C_0 \times {}^2C_2 = 4$ 2R, 1W, 0B: ${}^4C_2 \times {}^3C_1 \times {}^2C_0 = 18$ 2R, 0W, 1B: ${}^4C_2 \times {}^3C_0 \times {}^2C_1 = 12$ Number of selections $24 + 6 + 3 + 12 + 4 + 18 + 12 = 79$	1 – determining combinations involving 2 reds and 1 other colour 1 – determining combinations involving 2 blacks and 1 other colour 1 – summing all arrangements
	Total	**12 Marks**
3. (a)	$\frac{dy}{dx} - \frac{2}{x}y = 2x^3e^{2x}$ $I.F = e^{-\int \frac{2}{x} dx} = \frac{1}{x^2}$ $\left(\frac{1}{x^2}\right)\frac{dy}{dx} - \frac{2}{x^3}y = 2xe^{2x}$ $\int \left(\left(\frac{1}{x^2}\right)\frac{dy}{dx} - \frac{2}{x^3}y\right) dx = \int 2xe^{2x}\ dx$ $\frac{y}{x^2} = \int 2xe^{2x}\ dx$ $u = 2x \quad \rightarrow \quad du = 2$ $dv = e^{2x} \quad \rightarrow \quad v = \frac{1}{2}e^{2x}$ $\int 2xe^{2x}\ dx = xe^{2x} - \int e^{2x}\ dx$ $= xe^{2x} - \frac{1}{2}e^{2x} + c$ $\frac{y}{x^2} = xe^{2x} - \frac{1}{2}e^{2x} + c$ $y = x^3e^{2x} - \frac{1}{2}x^2e^{2x} + cx^2$ When $y = e^4, x = 2$ $e^4 = 2^3e^4 - \frac{1}{2}(2)^2e^4 + c(2)^2$ $e^4 = 8e^4 - 2e^4 + 4c$ $-5e^4 = 4c$	1 – determine the Integrating Factor 1 – exact differential of left hand side 1 – Integrating $2xe^{2x}$ by parts (S.O.I) 1 – Substituting expressions into $uv - \int vdu$ 1 – Integrating e^{2x} (omission of c accepted) 1 – Rearranging to make y the subject 1 – substituting values into general solution

	$c = -\frac{5}{4}e^4$ $y = x^3e^{2x} - \frac{1}{2}x^2e^{2x} - \frac{5}{4}x^2e^{2x}$	1 – correct value of c (68.2 accepted) 1 – Particular solution
3. (b)	$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 3x^2 + 5$ Auxiliary equation $u^2 + 5u + 6 = 0$ $(u + 2)(u + 3) = 0$ $u = -3, -2$ $y = Ae^{-3x} + Be^{-2x}$ Particular Integral Let $y = Dx^2 + Cx + E$ $\frac{dy}{dx} = 2Dx + C$ $\frac{d^2y}{dx^2} = 2D$ $2D + 5(2Dx + C) + 6(Dx^2 + Cx + E) = 3x^2 + 5$ $2D + 10Dx + 5C + 6Dx^2 + 6Cx + 6E = 3x^2 + 5$ $6Dx^2 + (10D + 6C)x + (5C + 2D + 6E) = 3x^2 + 5$ $6D = 3$ $D = \frac{1}{2}$ $10D + 6C = 0$ $C = -\frac{5}{6}$ $5C + 2D + 6E = 5$ $E = \frac{49}{36}$	1 – Auxiliary equation 1 – his complementary function 1 – correct trial equation for particular integral 1 – substituting expressions into original equation **1 mark each – his value for each constant**

	$y = \frac{1}{2}x^2 - \frac{5}{6}x + \frac{49}{36}$ General Solution $y = Ae^{-3x} + Be^{-2x} + \frac{1}{2}x^2 - \frac{5}{6}x + \frac{49}{36}$	1 – combining his complementary function and his particular integral
	Total	**17 Marks**
4. (i)	$10x + 15y + 5z = 550 \quad \rightarrow \quad 2x + 3y + z = 110$ $8x + 16y + 8z = 640 \quad \rightarrow \quad x + 2y + z = 80$ $10x + 10y + 10z = 700 \quad \rightarrow \quad x + y + z = 70$	**1 mark for each equation**
(ii)	$\begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 110 \\ 80 \\ 70 \end{pmatrix}$	**1 mark for each correct matrix**
(iii)	Using Row 3 $\lvert A\rvert = 1\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}$ $\lvert A\rvert = 1 - 1 + 1$ $\lvert A\rvert = 1$	1 – correct signs for each determinant 1 – correct use of the elements of the row or column as coefficients for the determinants 1 – correct cofactors 1 – correct evaluation of each determinant 1 – correct value for the determinant of A
(iv)	$B = \begin{pmatrix} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 & -1 \\ -2 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$ $B^T = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}$ $A^{-1} = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}$	1 – correct signs for each determinant **1 for each correct row or column not used in calculation of the determinant of A** 1 for correctly evaluating the determinants for the 2 rows or columns 1 – Transposing B 1 – the correct inverse

(v)	$\begin{pmatrix}2 & 3 & 1\\1 & 2 & 1\\1 & -1 & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}110\\80\\70\end{pmatrix}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1 & -2 & 1\\0 & 1 & -1\\-1 & 1 & 1\end{pmatrix}\begin{pmatrix}110\\80\\70\end{pmatrix}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}20\\10\\40\end{pmatrix}$	1 – attempting to multiply by A^{-1} 1 – the correct step up for multiplication (Inverse not written after $\begin{pmatrix}110\\80\\70\end{pmatrix}$) **1 for each correct answer**
	Total	**23 Marks**

					Alternative version of tree diagram for Q2(i)	

Question			Answer	Marks	Guidance	
2	(i)			G1 G1 G1 [3]	Do a vertical scan and give: First column Second column Final column Do not award if first two branches missing Branches two and three should come out of 'retest'	Allow labels such as A, R, F(Fail) etc All probabilities correct All probabilities correct All probabilities correct If any labels missing or incorrect allow max 2/3 Do not allow misreads here as all FT (eg 0.3 and 0.5 reversed)
2	(ii)		P(Accepted) = 0.2 + (0.3 × 0.2) + (0.3 × 0.3 × 0.4) = 0.2 + 0.06 + 0.036 = 0.296	M1 A1 [2]	For second or third product CAO	FT their tree provided correct numbers of terms and correct structure of 3, 3, 2 branches. Allow 37/125 oe
2	(iii)		P(At least one retest given accepted) $= \frac{\text{P(At least one retest and accepted)}}{\text{P(Accepted)}}$ $= \frac{(0.3 \times 0.2) + (0.3 \times 0.3 \times 0.4)}{0.296} = \frac{0.096}{0.296}$ = 0.324	M1 M1 A1 [3]	For numerator For denominator FT their 0.296 and 0.096 Allow 0.32 with working	FT their tree provided correct numbers of terms and correct structure of 3, 3, 2 branches. for both M1's Both must be part of a fraction Allow 12/125 oe Allow 12/37 oe

Q	Solution	Marks	Total	Comments
4	IF is $e^{\int -\frac{2}{x}\,dx}$	M1		Award even if negative sign missing
	$= e^{-2\ln(x)\ (+c)} = e^{\ln(x)^{-2}\ (+c)}$	A1		OE Condone missing c
	$= (k)x^{-2}$	A1F		Ft earlier sign error
	$x^{-2}\dfrac{dy}{dx} - 2x^{-3}y = 2xe^{2x}$			
	$\dfrac{d}{dx}\left(x^{-2}y\right) = 2x\,e^{2x}$	M1		LHS as $d/dx(y\times IF)$ PI
	$x^{-2}y = \int 2x\,e^{2x}\,dx$			
	$= \int x\ d(e^{2x}) = x\,e^{2x} - \int e^{2x}\,dx$	M1 A1		Integration by parts in correct dirn
	$x^{-2}y = xe^{2x} - \dfrac{1}{2}e^{2x}\ (+c)$	A1		ACF
	When $x = 2, y = e^4$ so $\dfrac{1}{4}e^4 = 2e^4 - \dfrac{1}{2}e^4 + c$	m1		Boundary condition used to find c after integration.
	$c = -\dfrac{5}{4}e^4$			
	$y = x^3e^{2x} - \dfrac{1}{2}x^2e^{2x} - \dfrac{5}{4}x^2e^4$	A1	9	Must be in the form $y = f(x)$
	Total		**9**	

(b)	Aux eqn $m^2 + 5m + 6 = 0$ $(m+3)(m+2) = 0$	**M1**		Correct factorising or correct substitution into quadratic formula OE on correct aux eqn. PI by correct values of 'm' seen/used.
	$(y_{CF} =)\ Ae^{-3x} + Be^{-2x}$	**A1**		
	Try $(y_{PI} =)\ ax^2 + bx + c$	**M1**		Correct form for y_{PI}. If other term(s) included, cand needs to show the corresponding coefficient is 0
	$(y'_{PI} =)\ 2ax + b;\quad (y''_{PI} =)\ 2a$			
	$2a + 5(2ax + b) + 6(ax^2 + bx + c) = 3x^2 + 5$	**dM1**		Substitution into DE, dep on previous **M** only. PI by at least two correct equations in next line provided previous M scored.
	$6a = 3;\quad 10a + 6b = 0;\quad 2a + 5b + 6c = 5$	**A1**		OE At least **two** correct, seen or used
	$a = \dfrac{1}{2};\quad b = -\dfrac{5}{6};\quad c = \dfrac{49}{36}$	**A1**		Seen or used; at least two correct
	$(y_{GS} =)\ Ae^{-3x} + Be^{-2x} + \dfrac{1}{2}x^2 - \dfrac{5}{6}x + \dfrac{49}{36}$	**A1**	**7**	ACF but must be exact
	Total		**12**	