CAPE PURE MATHEMATICS

UNIT 2

MODULE 1 Preview Test (2021)

1. Differentiate with respect to x

$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x}}$

$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x}}$

(a) $\ln(x^2 - 4x)$ [2]

(b) $\frac{x^2}{e^{2x+3}}$ [4]

(c) $x\sin^{-1}(x)$ [4]

Ans: (a) $\frac{2x-4}{x^2-4x}$ (b) $-\frac{2x(x-1)}{e^{2x+3}}$ (c) $\frac{x}{\sqrt{1-x}} + \sin^{-1} x$

2. Given that $\cos y = xy + y^2$, find $\frac{dy}{dx}$ in terms of x and y. [5]

Ans: $-\frac{y}{x+2y+\sin y}$

3. A curve is defined parametrically by the equations

$$x = t + \ln t\,, \quad y = t - \ln t$$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

Find the gradient of the curve at the point where $t = -2$. [5]

Ans: 3

4. Find the exact value of

$\int u\,dv = [uv]_1^2 - \int_1^2 v\,du$

$$\int_1^2 \underbrace{x^2}_{dv} \underbrace{\ln x}_{u}\, dx$$

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[5]

Ans: $\frac{8}{9}\ln 8 - \frac{7}{9}$

5. Using partial fractions, evaluate

$$\int \frac{2x}{(x-1)(2x+1)}\, dx$$

[7]

Ans: $\frac{2}{3}\ln|x-1| + \frac{1}{3}\ln|2x+1| + c$

$\ln\left|(x-1)^{2/3}(2x-1)^{1/3}\right| + c$

6. Use the substitution $u = \cos x$, or otherwise, to find the exact value of

$$\int_0^{\frac{\pi}{3}} \sin^5 x \cos^2 x \; dx$$

[6]

Ans: $\frac{617}{13440}$

7. Use the trapezium rule with 5 strips to evaluate

$f(x) = \sqrt{1+e^{-x}}$

$$\int_1^2 \sqrt{1+e^{-x}}\, dx$$

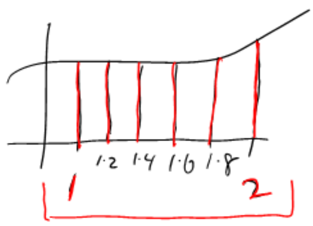
$y_0 = f(1) = \sqrt{1+e^{-1}} =$

$y_1 = f(1.2) = \sqrt{1+e^{-1.2}} =$

y_2

y_3

y_4



$\frac{2-1}{5} = 0.2$

[5]

Ans: 1.1101

$3i(a+bi)+5(a-bi)-14=2i$

$3ai+3bi^2+5a-5bi-14=2i$

$(3a$

8. Find the complex number z such that

$$3iz + 5z^* - 14 = 2i$$

Give your answer in the form $a + bi$, where a and b are real.

NB: z^* is the conjugate of z.

[6]

Ans: $4 + 2i$

9. (a) A circle C in the Argand diagram has equation

$$|z - 5 + i| = \sqrt{7}$$

Write down its radius and the complex number representing its centre. [2]

(b) A half – line L in the Argand diagram has equation

$$\arg(z + 3i) = \frac{3\pi}{4}$$

$\arg(-6+3i+3i)$

$\arg(-6+6i)$

Show that $z_1 = -6 + 3i$ lies on L. [3]

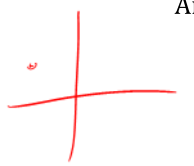
Ans: (a) Centre $5 - i$ radius $\sqrt{7}$

10. Use de Moivre's Theorem to find an expression for $\sin 5\theta$. [6]

Ans: $\sin 5\theta = 5\cos^4\theta \sin\theta - 10\cos^2\theta \sin^3\theta + \sin^5\theta$

$\arg(-6+6i) = \pi - \tan^{-1}\left(\frac{6}{6}\right)$

$= \frac{3\pi}{4}$



$$\int_0^{\pi/3} \sin^5 x \cos^2 x \quad dx$$

$$\int f'(x)[f(x)]^n \, dx = \frac{f(x)^{n+1}}{n+1} + c$$

$$= \int \sin x \, (\sin^2 x)(\sin^2 x) \cos^2 x \, dx$$

$$= \int \sin x \, (1-\cos^2 x)(1-\cos^2 x) \cos^2 x \, dx$$

$$= \int \sin x \, (1 - 2\cos^2 x + \cos^4 x) \cos^2 x \, dx$$

$$= \int \sin x \, (\cos^2 x - 2\cos^4 x + \cos^6 x) \, dx$$

$$= \int \sin x \, (\cos x)^2 dx - 2\int \sin x (\cos x)^4 dx + \int \sin x (\cos x)^6 \, dx$$

$$= -\int -\sin x \, (\cos x)^2 \, dx + 2\int -\sin x (\cos x)^4 \, dx - \int -\sin x (\cos x)^6 \, dx$$

$$= -\frac{\cos^3 x}{3} + 2\frac{\cos^5 x}{5} - \frac{\cos^7 x}{7}$$

$i = i \qquad i^3 = -i \qquad i^5 = i$

$i^2 = -1 \qquad i^4 = 1$

$(\cos\theta + i\sin\theta)^5$

$= \binom{5}{0}\cos^5\theta + \binom{5}{1}\cos^4\theta\,(i\sin\theta) + \binom{5}{2}\cos^3\theta\,(i\sin\theta)^2 + \binom{5}{3}\cos^2\theta\,(i\sin\theta)^3 + \binom{5}{4}\cos\theta\,(i\sin\theta)^4$

$+ \binom{5}{5}(i\sin\theta)^5$

$= 1\cos^5\theta + \underline{5\cos^4\theta(i\sin\theta)} + 10\cos^3\theta\,(-\sin^2\theta) + \underline{10\cos^2\theta(-i\sin^3\theta)} + 5\cos\theta\sin^4\theta + \underline{i\sin^5\theta}$

$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$