HARRISON COLLEGE INTERNAL EXAMINATION 2022
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 – TEST 1
1 hour 20 minutes

This examination paper consists of 14 pages.
This paper consists of 3 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

---

1. (a) (i)   Differentiate $f(x) = cos^{-1}3x$ where $-\frac{1}{3} < x < \frac{1}{3}$.   [2]

   (ii)   Differentiate $y = \frac{\ln 3x}{sin^{-1}x}$ [3]

PLEASE TURN OVER

(b) The curve C is defined parametrically by

$$x = t + \ln(t + 1), \quad y = 3te^{2t}$$

Find the equation of the tangent to the curve at the origin. [5]

(c) Consider the curve defined by $y^2 = \sin(xy)$, $y \neq 0$, show that $\dfrac{dy}{dx} = \dfrac{y\cos(xy)}{2y - x\cos(xy)}$.   [5]

(d) Let $f(x,y) = x^2y - 2x + y^3$,   find $\dfrac{\partial^2 f}{\partial x \partial y}$.   [2]

Total: 17 marks

PLEASE TURN OVER

2. (a) Find $\int \frac{1}{\sqrt{16-6x-x^2}}\, dx$ [3]

(b) Let $f(x) = \dfrac{6x^2 + 8x + 9}{(2-x)(3+2x)^2}$

    i.   Express $f(x)$ in partial fractions.   [5]

ii. Hence find $\int_{-1}^{0} f(x) \, dx$ giving your answer in the form $a + \frac{1}{2}\ln\left(\frac{b}{c}\right)$, where $a, b, c \in \mathbb{R}$. [5]

(c) It is given that for $n \geq 0$

$$I_n = \int_0^e x(\ln x)^n \, dx$$

(i) Prove that for $n \geq 1$, $\quad 2I_n = e^2 - nI_{n-1}$. [4]

(ii) Find the exact value of I_3 .   [4]

(d)   Use the trapezium rule with 4 strips to find an approximation to

$$\int_{-1}^{1} \sqrt{\ln(2 + x)} \, dx$$

giving your answer to 2 decimal places.   [4]

Total: 25 marks

3. (a) Showing all necessary working, express the complex number $\frac{2+3i}{1-2i}$ in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$.

Give the value of r and θ correct to 3 significant figures. [5]

(b)   The complex number $2 - i$ is denoted by u.

It is given that u is root of the equation $x^3 + ax^2 - 3x + b = 0$, where the constants a and b are real. Find the values of a and b. [4]

(c)   The complex number u is given by $u = -1 + (4\sqrt{3})i$.

Find the two square roots of u.

Give your answers in the form $a + ib$, where a and b are exact.   [5]

(d) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - 3 + 2i| = 1$. Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [4]

Total: 18 marks

END OF EXAMINATION