HARRISON COLLEGE INTERNAL EXAMINATION 2021

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 1 – TEST 2

Time: 1 hour and 20 minutes

<u>**NAME OF STUDENT:**</u>

<u>**SCHOOL CODE:**</u> 030014

<u>**DATE**</u>:

This examination paper consists of 9 printed pages and 1 blank page.

The paper consists of 4 questions.

The maximum mark for this examination is 60.

<u>INSTRUCTIONS TO CANDIDATES</u>

1. Write your name clearly in the space above.

2.   Answer **ALL** questions in the **SPACES PROVIDED**.

3. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use

    the extra page(s) provided. You must also write your name and candidate number clearly on any additional

    paper used.

4. Number your questions **carefully and identically to those on the question paper**.

5. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct

    to three (3) significant figures.

<u>EXAMINATION MATERIALS ALLOWED</u>

1. Mathematical formulae

2. Electronic calculator (non-programmable, non-graphical).

1. a) Prove that

$$\sin\left(\frac{\pi}{4} + \theta\right)\cos\left(\frac{\pi}{4} - \theta\right) + \cos\left(\frac{\pi}{4} + \theta\right)\sin\left(\frac{\pi}{4} - \theta\right) = 1$$

[3 marks]

b) Show that

$$4 + (\tan x - \cot x)^2 \equiv cosec^2 x + sec^2 x$$

[4 marks]

c) Find the general solution of the equation

$$6\cos^2 x + \sin x = 4$$

[7 marks]

d) i) Express $2\cos x - 7\sin x$ in the form $r\cos(x + \alpha)$ where $r > 0$ and $0^0 \le x \le 90^0$.

[4 marks]

ii) Hence, solve the equation $2\cos x - 7\sin x = -5$, for $0^0 \leq x \leq 360^0$.

[6 marks]

Total 24 marks

2. A circle, C, has equation $x^2 + y^2 + 2x - 8y = 152$.

i) Find the radius and the coordinates of the centre of the circle.

[4 marks]

ii)   Find the equation of the tangent to the circle at the point $P(4, 16)$.

[4 marks]

iii)   The point P is at one end of the diameter of the circle C. Find the coordinates of the point Q, at

the opposite end of the diameter from P.

[3 marks]

Total 11 marks

3. a) A curve, C, has parametric equations

$$x = \sin^2\theta \text{ and } y = \cos\theta, 0 \leq \theta < \frac{\pi}{2},$$

Find the Cartesian equation of C.

[3 marks]

b) Find the points of intersection of the curve

$$C_1 \text{ with parametric equations } x = 4t^2 \text{ and } y = 8t$$

and the line

$$L_1 \text{ with Cartesian equation } 3y + 16 = 4x$$

4. a) Find the angle between the lines with equations

$$r = \begin{pmatrix} 3 \\ -1 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} \text{ and } \frac{x+5}{2} = \frac{y-3}{1} = \frac{z+1}{-1}$$

[5 marks]

b) Find the point of intersection of the lines with equations

$$r_1 = \begin{pmatrix} 3 \\ 7 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix} \quad and \quad r_2 = \begin{pmatrix} -6 \\ 17 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

[6 marks]

c) Find the Cartesian equation of the plane passing through the point $A(4, 2, -7)$ and parallel to the plane with equation $3x + 5y - z = 7$.

[4 marks]

Total 15 marks

END OF TEST

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