HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2017
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 1 – TEST 1
1 hour 20 minutes

This examination paper consists of **2** printed pages.
This paper consists of **9** questions.
The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES
(i) Write your name clearly on each sheet of paper used
(ii) Answer **ALL** questions
(iii) Number your questions identically as they appear on the question paper and do **NOT write your solutions to different questions** beside each other
(iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED
(a) Mathematical formulae
(b) Scientific calculator (non-programmable, non-graphical)

1) Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are propositions, use the algebra of propositions to fully simplify $(\boldsymbol{a} \wedge \boldsymbol{b}) \vee (\sim \boldsymbol{a} \wedge \boldsymbol{b})$ [3]

Total: 3 marks

2) (a) (i) Express in terms of n, $\sum_{r=1}^{n+1}(2r)$. [3]
(ii) Given that $\sum_{r=1}^{n+1}(2r) = 42$, find the possible value(s) of n. [4]

(b) Determine the values of a and b such that $\dfrac{5\sqrt{3}-2}{3-\sqrt{3}} \equiv a + b\sqrt{3}$. [4]

Total: 11 marks

3) (i) Given $f(x) = x^3 + bx^2 + cx + 8$ is divisible by both $(x + 1)$ and $(x – 2)$, find the values of b and c. [4]

(ii) Hence, with these values of b and of c, solve $f(x) = 0$. [4]

Total: 8 marks

4) Prove by mathematical induction that $\sum_{r=1}^{n} \dfrac{1}{(4r-3)(4r+1)} = \dfrac{n}{4n+1} \ \forall\ n \in \boldsymbol{N}$. [6]

Total: 6 marks

P.T.O

5) (a) Solve for p, $9^{p^2} = 3^{p+3}$. [4]

(b) Solve for x the equation $e^x + 4e^{-x} = 4$, giving your answer in terms of *logs*. [4]

Total: 8 marks

6) The number of ants, N, on a tree at 1st March 2017 was 30 000.

The number of ants after n days was found to be directly proportional to $\left(\frac{1}{2}\right)^n$.

Calculate estimates of

(i) the number of ants after 5 days [2]
(ii) the day on which the population is expected to reach 200. [4]

Total: 6 marks

7) (a) The function f is defined by $f: x \to ln\,(x-4)$.

(i) Sketch the graph of f, showing clearly any intersection with the axes. [2]
(ii) Determine an expression for the inverse function, $f^{-1}(x)$. [3]

(b) The function g is defined by $g: x \to e^x + 3$.
Determine $gf(x)$, simplifying your answer. [2]

Total: 7 marks

8) Find the range of values of x for which $\frac{2x+3}{3x-2} + 2 \le 0$, $x \neq \frac{2}{3}$. [5]

Total: 5 marks

9) Solve for $x \in \boldsymbol{R}$, $2x^2 - |x| - 3 = 0$.

[6]

Total: 6 marks

End of Test

1) ***b***

2) (a) (i) $(n + 1)(n + 2)$

(ii) $n = 5$

(b) $a = \frac{3}{2}$, $b = \frac{13}{6}$

3) (i) $b = -5$, $c = 2$

(ii) $x = -1$, $x = 2$, $x = 4$

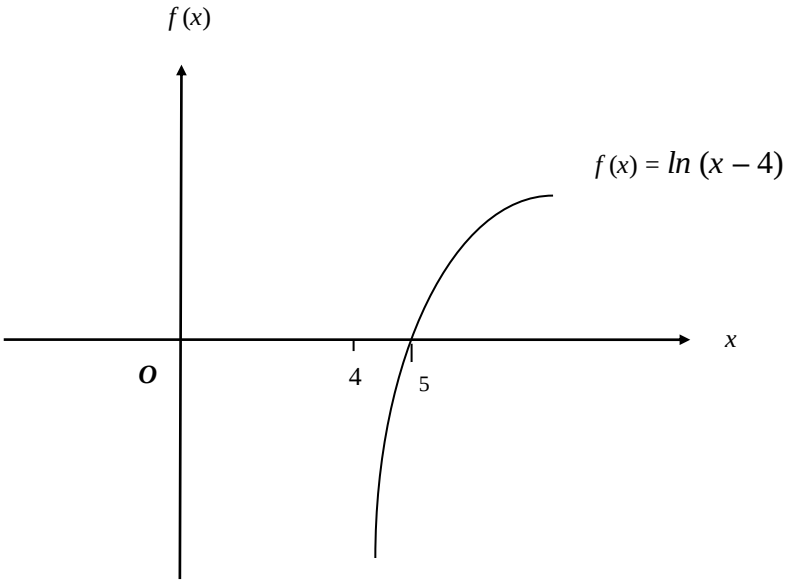
5) (a) $p = \frac{3}{2}$, $p = -1$

(b) $x = \ln 2$

6) $N = 937.5$ ants

(ii) 7.23 days = n

7) (a) (i)



(ii) $e^{x} + 4 = f^{-1}(x)$

(b) $g f(x) = x - 1$

8) $\frac{1}{8} \leq x < \frac{2}{3}$

9) Final solution $x = \pm \frac{3}{2}$