UNIT 1 TEST 3 (2015) PREVIEW

1 (a) $$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$$

$$= \lim_{x\to 9} \frac{\sqrt{x}}{(\sqrt{x} - \quad \sqrt{x}+3)}$$

$$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

(b) $$\lim_{x\to 0} \frac{\sin 5x}{3x}$$

$$= \lim_{x\to 0} \frac{\sin 5x}{5x} \quad \frac{5\cancel{x}}{3\cancel{x}}$$

$$= \lim_{x\to 0} \frac{\sin 5x}{5x} \cdot \frac{5}{3}$$

$$= \frac{5}{3} \lim_{x\to 0} \frac{\sin 5x}{5x}$$

$$= \frac{5}{3}$$

let $u = 5x$

as $x\to 0$ $\quad u\to 0$

so $$\lim_{x\to 0} \frac{\sin 5x}{5x}$$

$$= \lim_{u\to 0} \frac{\sin u}{u}$$

$$= 1$$

(c) For $f(x)$ to be continuous at $x=2$

$$f(2) = 2k+1 = \lim_{x\to 2} \frac{x^2+3x-10}{x-2}$$

$$= \lim_{x\to 2} \frac{(\cancel{x-2})(x+5)}{(\cancel{x-2})} = 2+5 = 7$$

$$2k+1 = 7$$

$$k = 3$$

1 (d) $f(x) = x^{-2}$

$f(x+h) = (x+h)^{-2}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{(x+h)^{-2} - x^{-2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x^2 - (x+h)^2}{(x+h)^2 x^2}}{h}$$

$$= \lim_{h \to 0} \frac{x^2 - (x^2 + 2xh + h^2)}{(x+h)^2 x^2} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-2x\cancel{h} - h^{\cancel{2}}}{(x+h)^2 x^2} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-2x - h}{(x+h)^2 x^2}$$

$$= \frac{-2x}{x^2 x^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

2. (a) (i) $f(x) = \sqrt{x^2-4} = (x^2-4)^{1/2}$

$$f'(x) = \frac{1}{2}(x^2-4)^{-1/2} \times 2x$$

$$= x(x^2-4)^{-1/2}$$

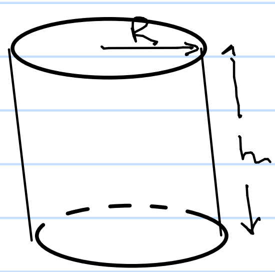
$$= \frac{x}{(x^2-4)^{1/2}} = \frac{x}{\sqrt{x^2-4}}$$

a (ii) $f(x) = \dfrac{3x}{\sin 2x}$

$$f'(x) = \frac{(\sin 2x)(3) - 3x(\cos 2x)(2)}{\sin^2 2x}$$

$$= \frac{3\sin 2x - 6x\cos 2x}{\sin^2 2x}$$

2 (b)



(i) Volume $= \pi R^2 h = 2000$

$$h = \frac{2000}{\pi R^2}$$

(ii) Surface area $= 2\pi R^2 + 2\pi R h$

$$A = 2\pi R^2 + 2\pi R\left(\frac{2000}{\pi R^2}\right)$$

$$A = 2\pi R^2 + \frac{4000}{R}$$

(iii) $\dfrac{dA}{dR} = 4\pi R - \dfrac{4000}{R^2} = 0$ for minimum area

$$4\pi R^2 - \frac{4000}{R^2} = 0$$

$$4\pi R^2 = \frac{4000}{R^2}$$

$$R^3 = \frac{4000}{4\pi} = \frac{1000}{\pi}$$

$$R = \sqrt[3]{\frac{1000}{\pi}} = \frac{10}{\sqrt[3]{\pi}} = 6.83 \text{ cm}$$

(c) (i) $x = 5t - 4$

$$\frac{dx}{dt} = 5$$

$$y = 1 - \frac{3}{t}$$

$$\frac{dy}{dt} = \frac{3}{t^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{1}{\frac{dx}{dt}}$$

$$= \frac{3}{t^2} \times \frac{1}{5} = \frac{3}{5t^2}$$

(ii) when $t = 1$ $\frac{dy}{dx} = \frac{3}{5}$ (gradient of tangent)

$$x = 5 - 4 = 1$$

$$y = 1 - \frac{3}{1} = -2$$

so equation of tangent is

$$y + 2 = \frac{3}{5}(x - 1)$$

$$\Rightarrow 5y + 10 = 3x - 3 \Rightarrow 3x - 5y - 13 = 0$$

3 (a) $\frac{dy}{dx} = (3x-4)^{-2}$

$y = \int (3x-4)^{-2}\, dx$

$y = \frac{(3x-4)^{-1}}{-3} + c$

since $(0,1)$ lies on the curve

$1 = +\frac{1}{12} + c$

$c = \frac{11}{12}$

so equation of curve is

$y = \frac{11}{12} - \frac{(3x-4)^{-1}}{3}$

$= \frac{11}{12} - \frac{1}{3(3x-4)}$

3 (b) (i) $\int_0^1 \cos(2-x)\, dx$

$= -\sin(2-x) \Big|_0^1$

$= [-\sin(2-1)] - [-\sin(2-0)]$

$= -0.841 - [-0.909]$

$= 0.068 \approx 0.07$

3 b (iii) $\int_1^2 2x(x^2-1)^3\,dx$

let $u = x^2 - 1$

$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

when $x = 1$ $u = 1^2 - 1 = 0$

$x = 2$ $u = 2^2 - 1 = 3$

on substitution integral becomes

$$\int_0^3 \cancel{2x}\,(u)^3\,\frac{du}{\cancel{2x}} = \int_0^3 u^3\,du$$

$$= \frac{u^4}{4}\Big|_0^3$$

$$= \frac{3^4}{4} - \frac{0^4}{4}$$

$$= \frac{81}{4}$$

(c) $A = \int_0^{\pi/3} \sin 3x + x\,dx$

$$= \frac{-\cos 3x}{3} + \frac{x^2}{2}\Big|_0^{\pi/3}$$

$$= \left[\frac{-\cos\left(3\frac{\pi}{3}\right)}{3} + \frac{\left(\frac{\pi}{3}\right)^2}{2}\right] - \left[\frac{-\cos 0}{3}\right]$$

$$= \frac{1}{3} + \frac{\pi^2}{18} + \frac{1}{3} = \frac{2}{3} + \frac{\pi^2}{16}$$

3 (d) $\frac{dy}{dx} = \frac{2x^3}{y}$

Separating the variables

$y\,dy = 2x^3\,dx$

$\int y\,dy = \int 2x^3\,dx$

$\frac{y^2}{2} = \frac{2x^4}{4} + C$

$\frac{y^2}{2} = \frac{x^4}{2} + C$

$y^2 = x^4 + C$

substitute $x = 1$ $y = 2$ to find c

$4 = 1 + c$

$c = 3$

$y^2 = x^4 + 3$