SBA UNIT 1 - TEST 2 (2014)

1 (a) $x = \sin t$ $\quad y = \tan t$

$x^2 = \sin^2 t$ $\quad y^2 = \dfrac{\sin^2 t}{\cos^2 t}$

$y^2 = \dfrac{\sin^2 t}{1 - \sin^2 t}$

$$\boxed{y^2 = \frac{x^2}{1 - x^2}}$$

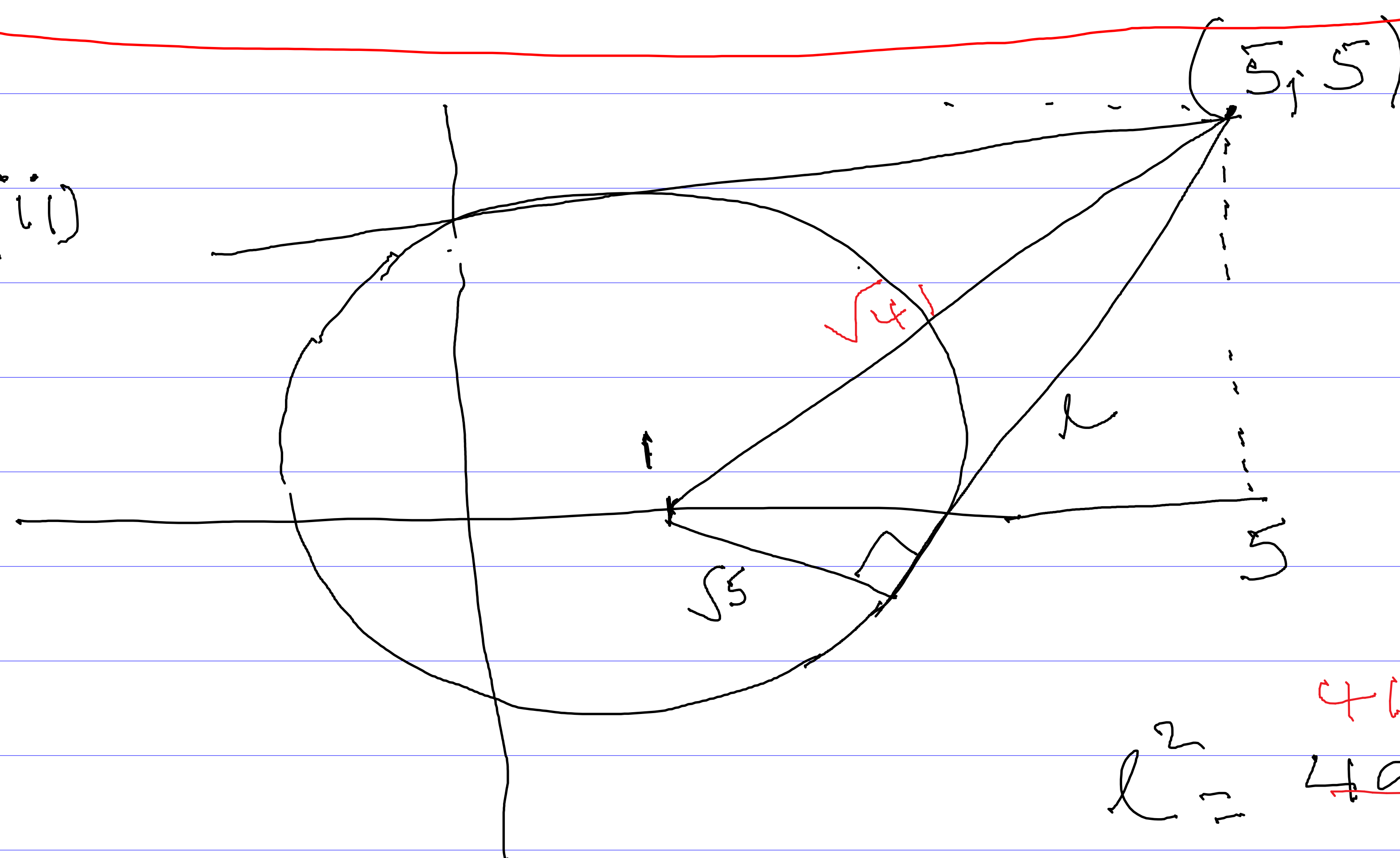
(b) $x^2 - 2x + y^2 - 4 = 0$

$[x^2 - 2x + (-1)^2] + y^2 - 4 - (-1)^2 = 0$

$(x-1)^2 + y^2 - 5 = 0$

centre of circle is $(1, 0)$
radius $= \sqrt{5}$

b(ii)



length from centre to $(5,5)$

$= \sqrt{5^2 + 4^2}$

$= \cancel{\sqrt{49}}$ $\sqrt{41}$

$41 - 5$

$l^2 = \cancel{49 - 5 = 44}$ 36

$\cancel{l = \sqrt{44}}$

$L = \sqrt{36} = 6$

1 (b) (iii)

C_1 $\quad x^2 + y^2 - 2x - 4 = 0$

C_2 $\quad x^2 + y^2 + 2x + 4y - 4 = 0$

$$4x + 4y = 0$$

$$x = -y$$

$$x^2 + x^2 - 2x - 4 = 0$$

$$2x^2 - 2x - 4 = 0$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$x = 2$ or $x = -1$

$y = -2$ $\qquad y = 1$

check C_1: $4 + 4 - 4 = 4$ ✓

C_2 $\quad 4 + 4 + 4 - 8 = 4$ ✓

$\boxed{\text{Coordinates of } P \text{ and } Q \text{ are } (2, -2) \text{ and } (-1, 1)}$

2 (a) $\dfrac{\cot x}{\sec x}$

$= \dfrac{\cos x}{\sin x} \cdot \cos x$

$= \dfrac{\cos^2 x}{\sin x} = \dfrac{1 - \sin^2 x}{\sin x}$

$= \dfrac{1}{\sin x} - \sin x$

$= \operatorname{cosec} x - \sin x$

2(b) $2\cos^2\theta - \sin\theta - 1 = 0$

$2(1 - \sin^2\theta) - \sin\theta - 1 = 0$

$2 - 2\sin^2\theta - \sin\theta - 1 = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

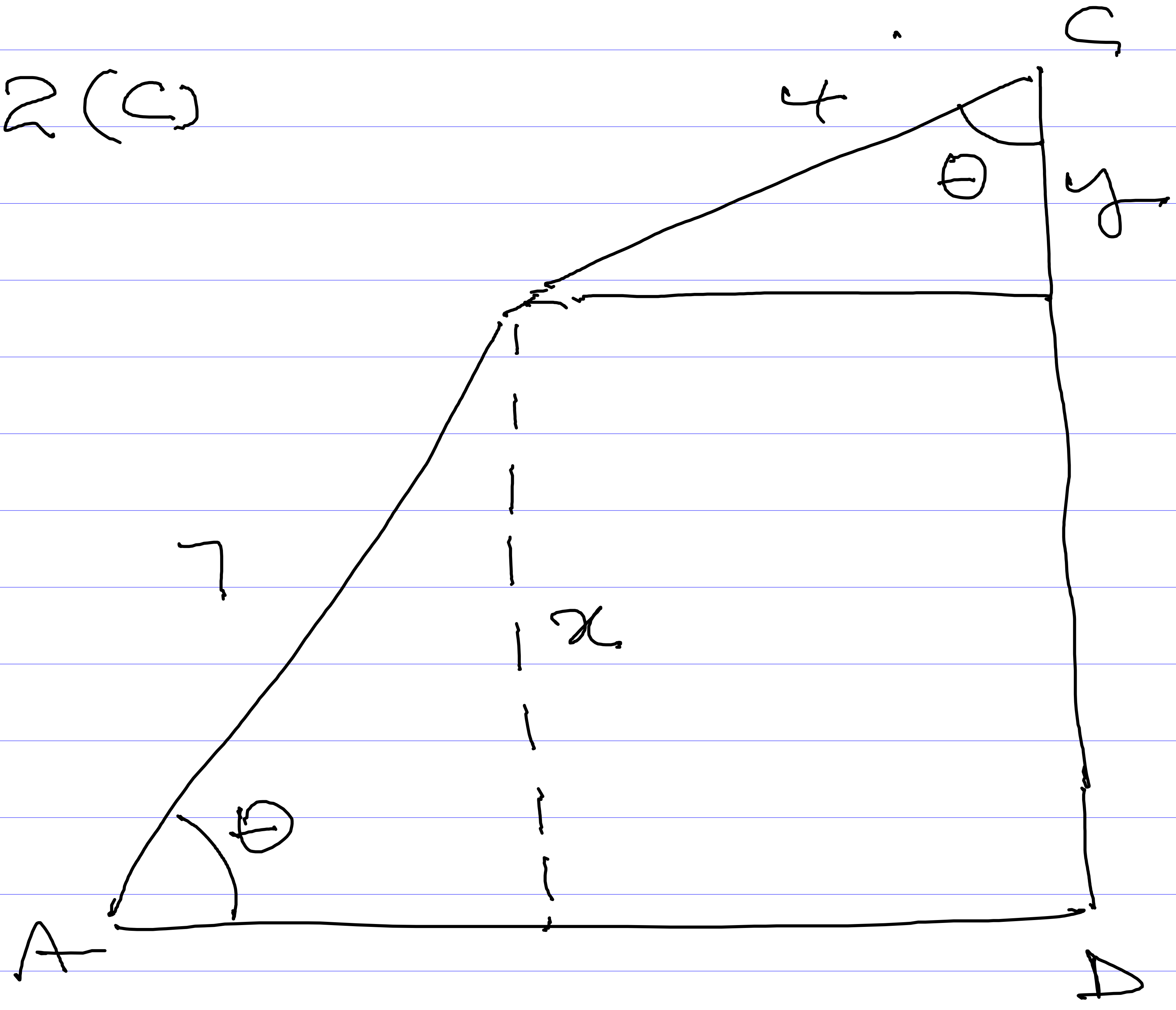
$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \dfrac{1}{2}$ $\qquad$ $\sin\theta = -1$

$\alpha = 30^\circ$ $\qquad$ $\theta = -90^\circ$

$\boxed{\theta = 180n + (-1)^n 30 \qquad \theta = 180n - (-1)^n 90}$

2(c)



$CD = x + y$

$\sin\theta = \frac{x}{7} \Rightarrow x = 7\sin\theta$

$\cos\theta = \frac{y}{4} \Rightarrow y = 4\cos\theta$

$CD = 4\cos\theta + 7\sin\theta$

(ii) $R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = 4 \qquad R\sin\alpha = 7$

$R = \sqrt{4^2 + 7^2} = \sqrt{16 + 49} = \sqrt{65}$

$\tan\alpha = \frac{7}{4} \Rightarrow \alpha = 60.26^{\circ}$

$= 1.05^{c}$

$4\cos\theta + 7\sin\theta =$

$$\boxed{\sqrt{65}\cos(\theta - 1.05)}$$

(iii) maximum value of

$$\boxed{CD = \sqrt{65}}$$

occurs at $\boxed{\theta = 1.05^c}$.

(iv) $BX = 4\sin\theta$

$AY = 7\cos\theta$

$$\text{Area} = 4\sin\theta \cdot 7\sin\theta + \frac{7\cos\theta \cdot 7\sin\theta}{2} + \frac{4\sin\theta \cdot 4\cos\theta}{2}$$

$$= 28\sin^2\theta$$

$$+ \frac{49\sin\theta\cos\theta}{2} + \frac{16\sin\theta\cos\theta}{2}$$

$$= 28\sin^2\theta + \frac{49\times 2\sin\theta\cos\theta}{4} + \frac{4\times 2\sin\theta\cos\theta}{1}$$

$$= \frac{112\sin^2\theta + 49\sin 2\theta + 16\sin 2\theta}{4}$$

$$= \frac{112\sin^2\theta + 65\sin 2\theta}{4}$$

3. $\vec{AB} = \overline{OB} - \overline{OA}$

$$= \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

$$\boxed{\overline{AB} = -i - j - k}$$

$$\overline{BC} = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$$\boxed{\overline{BC} = -i + 2j + k}$$

(ii) $r = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$

(iii) $a \cdot b = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 6 + 2 + 12 = 20$

$|a| = \sqrt{9+4+16} = \sqrt{29}$

$|b| = \sqrt{4+1+9} = \sqrt{14}$

$\cos\theta = \dfrac{20}{\sqrt{29}\sqrt{14}}$

$\boxed{\theta = 69.8^\circ}$

(b) (i) $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} = -1 - 2 + 3 = 0$

$\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = -1 + 4 - 3 = 0$

(ii) $r \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = 3 + 4 - 12$

$= -5$

$\boxed{r \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = -5}$