HARRISON COLLEGE INTERNAL EXAMINATION, MARCH 2017

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 1 – TEST 2

Time: 1 Hour & 20 minutes

This examination paper consists of 3 printed pages.

The paper consists of 8 questions.

The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Number your questions carefully and do **NOT** write your solutions to different questions bedside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Electronic calculator (non-programmable, non-graphical)

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1. Solve, for $-\pi \leq \theta \leq \pi$, the equation $3tan^2\theta + 4sec\theta = 1$ [7]

2. Prove that $sec2A + tan2A \equiv \frac{\cos A + sinA}{cosA - \sin A}$. [5]

3. Given that $cosecA = \frac{13}{5}$ and $cosB = \frac{3}{5}$, where $A\ and\ B$ are both acute angles, find the exact value of $cos(A + B)$. [6]

4. Express $5cosx - 3sinx$ in the form $Rcos(x + \alpha)$, where $R > 0\ and\ 0^\circ \leq \alpha \leq 90^\circ$, giving the exact value of R and the values of α correct to 1 decimal place.

   Hence solve the equation $5cosx - 3sinx = 4\ for\ 0^\circ \leq x \leq 360^\circ$. [6]

5. i) Show that the circle, C_1, with equation $x^2 + y^2 - 6x - 4y + 9 = 0$ touches the $x - axis$ and that the circle, C_2, with equation $x^2 + y^2 - 2x - 6y + 9 = 0$ touches the $y - axis$. [5]

   ii) Find the coordinates of the points of intersection of the two circles. [5]

   iii) Find the equation of the line passing through the two points of intersection. [3]

6. i) Determine the vector equation of the line joining the points $(2, 4, 4)\ and\ (3, 3, 5)$. [3]

   ii) Prove that the straight line with equation $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ intersects the line from part i).

   [3]

   iii) Determine the angle between the two lines. [3]

7. The position vectors of three points A, B and C on a mountain slope are

$$\boldsymbol{a} = 4\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}, \boldsymbol{b} = -2\boldsymbol{i} + 26\boldsymbol{j} + 11\boldsymbol{k}, \boldsymbol{c} = 16\boldsymbol{i} + 17\boldsymbol{j} + 2\boldsymbol{k}$$

   where the units are metres.

   i) Find the distance between the points $A\ and\ B$. [3]

   ii) Show that the vector $2\boldsymbol{i} - 3\boldsymbol{j} + 7\boldsymbol{k}$ is perpendicular to $\overrightarrow{AB}$ and also perpendicular to $\overrightarrow{AC}$. Hence find the equation of the plane of the mountain slope. [3]

   An overhead cable lies along *DEF*, where *D* and *E* have position vectors

   $\boldsymbol{d} = 130\boldsymbol{i} - 40\boldsymbol{j} + 20\boldsymbol{k}\ and\ \boldsymbol{e} = 90\boldsymbol{i} - 20\boldsymbol{j} + 15\boldsymbol{k}$, and *F* is a point on the mountain slope.

   iii) Find the equation of the straight line *DE*. [3]

   iv) Find the size of the acute angle between the cable and the slope. [5]