HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2022
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 – TEST 2
1 hour 20 minutes

This examination paper consists of 3 pages.
This paper consists of 7 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write in ink.
2. Write your name clearly on each sheet of paper used.
3. Answer **ALL** questions.
4. Do **NOT** do questions beside one another.
5. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 2,$$

$$u_{n+1} = bu_n - 3, \qquad n > 1$$

where b is a constant.

a) Find an expression for u_2 in terms of b. [1]

b) Show that $u_3 = 2b^2 - 3b - 3$. [2]

Given that $u_3 = 32$,

c) find the possible values of b. [3]

Total: 6 marks

2. Jaina saves money over a period of 200 weeks. She saves \$5 in week 1, \$7 in week 2, \$9 in week 3, and so on until week 200. Her weekly savings form an arithmetic sequence.

a) Find the amount she saves in week 200. [3]

b) Calculate her total savings over the complete 200 week period. [3]

Total: 6 marks

3. A series of positive integers $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 6 \text{ and } u_{n+1} = 6u_n - 5, \text{ for } n \geq 1.$$

Prove by induction that $u_n = 5 \times 6^{n-1} + 1$, for $n \geq 1$.

Total: 8 marks

4. a) Express

$$\frac{2}{(r+1)(r+3)}$$

in partial fractions. [4]

b) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} = \frac{n(an+b)}{6(n+2)(n+3)}$$

where a and b are constants to be found. [7]

c) Find the value of

$$\sum_{r=21}^{30} \frac{2}{(r+1)(r+3)}$$

to 5 decimal places. [3]

Total: 14 marks

5. a) Write down and simplify the first three non-zero terms of the Maclaurin series for $ln(1+3x)$. [3]

b) Hence find the first three non-zero terms of the Maclaurin series for $e^x ln(1+3x)$, simplifying the coefficients. [4]

Total: 7 marks

6. a) Use the binomial series to expand $\frac{1}{\sqrt{4-3x}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term. [5]

b) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+8}{\sqrt{4-3x}}$ as a series in ascending powers of x. [4]

Total: 9 marks

7.

$$f(x) = 4\cos x + e^{-x}$$

a) Show that the equation $f(x) = 0$ has a root α between 1.6 and 1.7 [4]

b) Use linear interpolation once, to obtain an approximation to α. Give your answer to 2 decimal places. [2]

c) Taking 1.6 as your first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 significant figures. [4]

Total: 10 marks

END OF EXAMINATION