HARRISON COLLEGE INTERNAL EXAMINATION 2021 PREVIEW

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 2 – TEST 2

Time: 1 hour and 20 minutes

1. a) In the expansion of $(3 + px)^6$, the coefficient of x^3 is thirty times the coefficient of the x^5 term. Find the possible values of p.

[5 marks]

$[p = \pm 1]$

b) i) Expand $\sqrt[4]{(1 - 2x)}$ in ascending powers of x up to and including the term in x^3 and state the values of x for which the expansion is valid.

[5 marks]

$$\left[1 - \frac{1}{2}x - \frac{3}{8}x^2 - \frac{7}{16}x^3 \qquad valid\ for - \frac{1}{2} < x < \frac{1}{2}\right]$$

ii) Use your expansion from i) to find an approximation for $\sqrt[4]{0.998}$ to 6 decimal places.

[3 marks]

$[0.999500]$

Total 13 marks

2. Find the Taylor series expansion for $f(x) = \sqrt{8 + e^x}\ about\ a = 0, x \in \mathbb{R}$, in ascending powers of x, up to and including the term in x^2. Express each coefficient in its simplest form.

[8 marks]

$$\left[3 + \frac{1}{6}x + \frac{17}{216}x^2 + \cdots\right]$$

3. A sequence is defined by

$$u_1 = \frac{3}{4} \quad and \quad u_{n+1} = \frac{3}{4 - u_n} \; for \; n = 1, 2, 3, \ldots$$

i) Calculate u_2.

[2 marks]

$$\left[\frac{12}{13}\right]$$

ii) Prove by mathematical induction that, for $n \geq 1$,

$$u_n = \frac{3^{n+1} - 3}{3^{n+1} - 1}$$

[8 marks]

Total 10 marks

4. i) Show that

$$\frac{1}{r} - \frac{1}{r+2} \equiv \frac{2}{r(r+2)}$$

[2 marks]

ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{2}{r(r+2)}$$

[6 marks]

$$\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}$$

iii) Find the value of N, given that

$$\sum_{r=N+1}^{\infty} \frac{2}{r(r+2)} = \frac{13}{42}$$

[6 marks]

Total 14 marks

5. a) i) Show that the equation

$$2x^5 + x^3 - 1 = 0$$

has a root between 0 and 1.

[3 marks]

ii) Use linear interpolation once, starting with the interval in a) i), to give an approximate value of this root.

[2 marks]

$$\left[\frac{1}{3}\right]$$

b) It is known that the function

$$f(x) = \sin x - \ln x$$

has a root α in the interval $[2.2\ ,\ 2.3]$.

i) Find $f'(x)$.

[2 marks]

$$\left[\cos x - \frac{1}{x}\right]$$

i) Using $x_0 = 2.3$ as a first approximation to α, apply the Newton-Raphson procedure twice to $f(x)$ to find a third approximation to α, giving your answer to 3 decimal places.

[7 marks]

$$[x_2 = 2.219]$$

Total 14 marks