HARRISON COLLEGE INTERNAL EXAMINATION 2021

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 2 – TEST 2

Time: 1 hour and 20 minutes

NAME OF STUDENT: _____

SCHOOL CODE: 030014

DATE: _____

This examination paper consists of 10 printed pages, including 1 blank page.

The paper consists of 5 questions.

The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly in the space above.
2. Answer **ALL** questions in the **SPACES PROVIDED**.
3. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided. You must also write your name and candidate number clearly on any additional paper used.
4. Number your questions **carefully and identically to those on the question paper**.
5. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Electronic calculator (non-programmable, non-graphical).

1. a) In the expansion of $(3 + px)^6$, the coefficient of x^4 is four times the coefficient of the x^2 term. Find the possible values of p.

[5 marks]

b) i) Expand $\sqrt[3]{(1 + 2x)}$ in ascending powers of x up to and including the term in x^3 and state the values of x for which the expansion is valid.

[5 marks]

ii) Use your expansion from i) to find an approximation for $\sqrt[3]{1.1}$ to 5 decimal places.

[3 marks]

Total 13 marks

2. Find the Maclaurin series expansion for $f(x) = \sqrt{8 + e^x}, x \in \mathbb{R}$, in ascending powers of x, up to and including the term in x^2. Express each coefficient in its simplest form.

[8 marks]

Total 8 marks

3. A sequence is defined by

$$u_1 = 2 \quad and \quad u_{n+1} = \frac{u_n}{1 + u_n}$$

i) Calculate u_3.

[2 marks]

ii) Prove by mathematical induction that, for $n \geq 1$,

$$u_n = \frac{2}{2n - 1}$$

[8 marks]

Total 10 marks

4. i) Show that

$$\frac{1}{r} - \frac{1}{r+2} \equiv \frac{2}{r(r+2)}$$

[2 marks]

ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{2}{r(r+2)}$$

[6 marks]

iii) Find the value of N, given that

$$\sum_{r=N+1}^{\infty} \frac{2}{r(r+2)} = \frac{11}{30}$$

[6 marks]

Total 14 marks

5. a) i) Show that the equation

$$x^3 + x - 7 = 0$$

has a root between 1.6 and 1.8.

[3 marks]

ii) Use linear interpolation once, starting with the interval in a) i), to give this root two decimal places.

[2 marks]

b) It is known that the function

$$f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20$$

has a root α in the interval $[1.1,\ 1.2]$.

i) Find $f'(x)$.

[3 marks]

ii) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure twice to $f(x)$ to find a third approximation to α, giving your answer to 3 significant figures.

[7 marks]

Total 15 marks

END OF TEST

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