

HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2021

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT **PREVIEW**

PURE MATHEMATICS

UNIT 1 – TEST 3

1 hour 30 minutes

This examination paper consists of 2 printed pages.

This paper consists of 3 questions.

The maximum mark for this examination is 60.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae

2. Scientific calculator (non-programmable, non-graphical)

1. (a) Find:

i. $\lim_{n \rightarrow 5} \frac{n^2 - 25}{n - 5}$ **Ans. 10** **[3]**

ii. $\lim_{x \rightarrow 0} \frac{\sin 4x}{3x}$ **Ans. $\frac{4}{3}$** **[5]**

(b) Given that $\lim_{x \rightarrow -2} \{4f(x)\} = 5$, evaluate $\lim_{x \rightarrow -2} \{f(x) + 2x\}$ **[5]**

Ans. $-\frac{11}{4}$

2. (a) Differentiate with respect to x , from first principles, the function,

$f(x) = 2x^2 - x + 5.$ **[5]**

Ans. $4x - 1$

(b) The equation of a curve is given to be $y = \frac{2x^3 + 1}{x^3 - 4}$, show that $\frac{dy}{dx} = \frac{-27x^3}{(x^3 - 4)^2}$. **[4]**

(c) Two parametric equations are given to be defined as **[6]**

$x = a \cos^2 \theta$, $y = a \sin^2 \theta$ where $a \in \mathbb{R}$, find the gradient $\frac{dy}{dx}$ in terms of θ .

Ans. -1

(d) The equation of a curve is given as $f(x) = 2x^3 - 3x^2 - 12x + 3$.

i. Determine f' . [1]

Ans. $6x^2 - 6x - 12$

ii. Determine f'' . [1]

Ans. $12x - 6$

iii. Find the coordinates of the stationary points and determine their nature. [10]

Ans. $(-1, 10) - \text{max}; (2, -5) - \text{min}$

3. (a) Use an appropriate substitution to find $\int x\sqrt{x^2 + 6} dx$. [4]

Ans. $\frac{1}{3}\sqrt{(x^2 + 6)^3}$

(b) Travis is inflating a spherical balloon at the rate $10 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of increase of the surface area of the balloon when the radius is 4 cm . (For a sphere, $V = \frac{4}{3}\pi r^3, A = 4\pi r^2$) [6]

Ans. $5 \text{ cm}^2 \text{ s}^{-1}$

(c) Sketch the graph $y = x^2 + x$ [5]

ii. Find the total area bounded by the curve in part (c), the x -axis and the lines $x = -1$ and $x = 3$. [5]

Ans. $13 \frac{2}{3} \text{ units}^2$