HARRISON COLLEGE INTERNAL EXAMINATION, March 2020
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS
UNIT 2 - TEST 1

TIME: 1 Hour & 20 minutes

This examination paper consists of 2 printed pages.
The paper consists of 3 questions.
The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Number your questions carefully and do **NOT** write your solutions to different questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Electronic calculator (non-programmable, non-graphical)

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1. (a) The complex numbers z and w are given by $z = 2 + 3i$ and $w = -1 - 6i$ respectively. Find.

(i) $2z - w$ [1]

(ii) $|2z - w|$ [2]

(iii) $\arg(2z - w)$ [2]

(iv) $\frac{z}{w}$ giving your answer in the form $x + iy$ [4]

(b) (i) Express $\sin n\theta$ and $\cos n\theta$ in terms of $e^{in\theta}$ and $e^{-in\theta}$. [2]

(ii) Hence show that

$$sin^4\theta = \frac{1}{8}(cos4\theta - 4cos2\theta + 3)$$ [5]

TOTAL 16 marks

2. (a) Find $\frac{dy}{dx}$ when

(i) $y = e^{\sqrt{x}} + cos^{-1}(2x)$ [3]

(ii) $y = \frac{\ln(x^2)}{sin^{-1}x}$ [3]

(b) Find the gradient of the curve $3x^2 + 2xy + (lny)^2 = 16$ at the point (2, 1). [4]

(c) A curve is defined by the parametric equations

$$y = \frac{t}{2t+3} \text{ and } x = e^{-2t}$$

Find the gradient of the curve at the point for which $t = 0$. [5]

(d) Let $f(x,y) = (x^2 + y^2)^5 + \ln(xy)$, find $\frac{\partial^2 f}{\partial x \partial y}$ [2]

TOTAL 17 marks

3. (a) (i) Express $f(x) = \frac{10x+9}{(2x+1)(2x+3)^2}$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 f(x)dx = \frac{1}{2}ln\left(\frac{9}{5}\right) + \frac{1}{5}$. [5]

(b) Using the substitution $u = x^2$, find

$\int_1^2 \frac{4x}{1+x^4}\, dx$ (give your answer to 2 decimal places) [5]

(c) It is given that for $n \geq 0$, $I_n = \int_0^{\frac{1}{2}} (1-2x)^n e^x \, dx$

(i) Show that for $n \geq 1$

$$I_n = 2nI_{(n-1)} - 1$$ [4]

(ii) Find the exact value of I_3. [4]

(d) Use the trapezium rule with 4 trapezia of equal width to estimate the value of

$\int_0^4 \frac{1}{1+\sqrt{x}}\, dx$. Give your answer to 2 decimal places. [4]

TOTAL 27 marks

END OF EXAMINATION