CAPE UNIT I, Test 3, 2020, Preview

1. a) Determine

$$\lim_{x \to -2} \frac{x+2}{2x^3 - 8x}$$ [4]

b) Differentiate $f(x) = x^2 + 5x - 7$, using first principles. [6]

c) The function $f(x)$ is defined by $f(x) = \begin{cases} x^3 & x \leq 2 \\ ax^2 & x > 2 \end{cases}$

Find i) $\lim_{x \to 2^-} f(x)$ [1]

ii) the value of a for which the function $f(x)$ is continuous [3]

Total 14 marks

2. A curve C has equation

$$y = \frac{x^2}{x+2}, x \neq -2$$

The point P on C has x-coordinate 2. Find an equation of the normal to the curve C at P in the form $ax + by + c = 0$, where a, b and c are integers. [7]

Total 7 marks

3. A curve has equation

$$y = \frac{x}{1+x^2}.$$

a) Use calculus to find the coordinates of the turning points of the curve. [5]

b) Show that

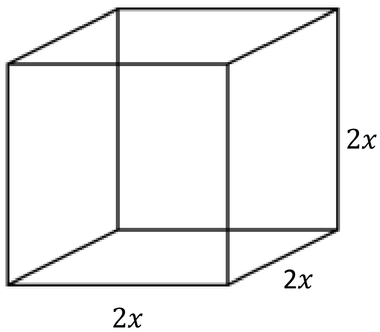
$$\frac{d^2y}{dx^2} = \frac{2x(x^2-3)}{(1+x^2)^3}$$

[4]

c) Determine the nature of each of the turning points. [4]

Total 13 marks

4. The figure below shows a metal cube which is expanding uniformly as it is heated. At time t seconds, the length of each edge of the cube is $2x$ cm, and the volume of the cube is V cm^3.



a) Show that $\frac{dV}{dx} = 24x^2$ [1]

b) Given that the volume, V cm^3, increases at a constant rate of 0.144 cm^3s^{-1}, find

i) $\frac{dx}{dt}$, when $x = 2$ [3]

ii) The rate of increase of the total surface area, A, of the cube, in cm^2s^{-1}, when $x = 2$. [4]

Total 8 marks

5. a) Use calculus to find the value of

$$\int_{-1}^{1} \left(\frac{2x^5 - 3x^4 + 2x^3}{x^2}\right) dx$$

[5]

b) Find the value of k if $\int_k^0 (x+4)^2 dx = \frac{64}{3}$. [5]

Total 10 marks

6 a) By using the substitution $U = sinx + cosx$, show that

$$\int (sinx - cosx)\,(sinx + cosx)^7 dx = -\frac{1}{8}(sinx + cosx)^8 + c$$

[5]

b) Hence, find the EXACT value of

$$\int_0^{\frac{\pi}{4}} (sinx - cosx)\,(sinx + cosx)^7 dx$$

[4]

Total 9 marks

Answers

1. a) $\frac{1}{16}$ b) $2x + 5$ c) i) 8 ii)$a = 2$
2. $4x + 3y - 11 = 0$
3. a) $\left(1, \frac{1}{2}\right)$ and $\left(-1, -\frac{1}{2}\right)$ b) Proof c) $maximum\left(1, \frac{1}{2}\right)$ $and\ minimum\left(-1, -\frac{1}{2}\right)$
4. a) Proof b) i) $0.0015\ cms^{-1}$ ii) $0.144\ cms^{-1}$
5. a)-2 b) $k = -4$
6. a) Proof b) $-\frac{15}{8}$