CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 2 – TEST 2

Time: 1 hour and 20 mimutes

This examination paper consists of 9 printed pages.

The paper consists of 6 questions.

The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your answers in the spaces provided.
2. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided. You must also write your name and candidate number clearly on any additional paper used.
3. Answer **ALL** questions.

4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Electronic calculator (non-programmable, non-graphical).

1. The population of a town is $25\ 000$ at the end of Year 1.

   A model predicts that the population of the town will increase by 3% each year.

   a) Determine the predicted population at the end of Year 2. [1]

   The model predicts that Year N will be the first year in which the population of the town exceeds 40 000.

   b) Show that $(N-1)\log 1.03 > \log 1.6$ [4]

   c) Find the value of N. [2]

At the end of each year, a donation of \$1 will be made to a charity fund on behalf of each member of the population. Assuming the population model,

d) Find the total amount that will be given to the charity fund for the 10 years from the end of Year 1, to the end of year 10, giving your answer to the nearest \$1 000. [3]

Total 10 marks

2. i) Show that

$$\frac{r}{r+1} - \frac{r-1}{r} \equiv \frac{1}{r(r+1)}$$

[2]

ii) Hence, using the method of differences, find an expression in terms of n, for

$$\frac{1}{2}+\frac{1}{6}+\frac{1}{12}\ldots+\frac{1}{n(n+1)}.$$

[5]

iii) Hence show that

$$\sum_{r=n+1}^{\infty}\frac{1}{r(r+1)}=\frac{1}{n+1}$$

[3]

Total 10 marks

3. A series of positive integers u_1, u_2, u_3 ... is defined by

$$u_1 = 6 \; and \; u_{n+1} = 6u_n - 5, for \; n \geq 1.$$

Prove by mathematical induction that $\boldsymbol{u_n} = (\mathbf{5} \times \mathbf{6}^{\boldsymbol{n-1}}) + \mathbf{1}$, for $n \geq 1$. [8]

Total 8 marks

4.

$$f(x) = x^3 - \frac{7}{x} + 2, \quad x > 0$$

a) Show that the equation $f(x) = 0$ has a root in the interval $[1.4, 1.5]$ [3]

b) Using $x_1 = 1.45$ as a first approximation to the real root α, apply the Newton-Raphson procedure once to find a second approximation to α, giving your answer to 3 significant figures. [5]

c) Starting with the interval $[1.4,\ 1.5]$, use the interval bisection method twice to obtain an approximation, giving your answer to 3 significant figures. [3]

Total 10 marks

5. a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, |x| < \frac{8}{3},$$

in ascending powers of x, up to and including the term in x^3, giving each term as a simplified fraction.

[5]

b) Use your answer with a suitable value of x, to obtain an approximation to $\sqrt[3]{7.7}$. Give your answer to 7 decimal places. [3]

Total 8 marks

6. a) Given $f(x) = e^{2x}$, obtain the Maclaurin expansion for $f(x)$ up to, and including the term in x^3.

[2]

b) On a suitable domain, let $g(x) = \tan x$

i) Show that the third derivative of $g(x)$ is given by

$$g'''(x) = 2sec^4x + 4tan^2xsec^2x$$

[4]

ii) Hence obtain the Maclaurin expansion for $g(x)$ up to and including the term in x^3. [4]

c) Hence, or otherwise, obtain the Maclaurin expansion for $e^{2x} \tan x$ up to and including the term in x^3.

[3]

13 marks

END OF TEST