HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2019

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 2 – TEST 2

1 hour 20 minutes

This examination paper consists of **3** printed pages.

This paper consists of **8** questions.

The maximum mark for this examination is **60**.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Scientific calculator (non-programmable, non-graphical)

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1. The first term of an arithmetic series is 1. The common difference of the series is 6.

   a) Find the tenth term of the series.   [2]

   b) The sum of the first n terms of the series is 7400.

      i)   Show that $3n^2 - 2n - 7400 = 0$.   [3]

      ii)   Find the value of n.   [2]

                                                                             Total 7 marks

2. A savings account pays 4.5% compound interest. A deposit of \$5 000 is invested in this scheme at the start of each year.

   a) Determine the amount of money in the account at the start of the third year **after** the annual deposit has been made.   [2]

   b) Find the amount in the savings account at the start of the 15th year **after** the annual deposit has been made.   [5]

                                                                             Total 7 marks

3. a) Show that the equation $e^x + x - 3 = 0$ has a root between 0 and 1.   [2]

   b) With a first approximation of 0.5, use the Newton-Raphson method to solve the equation $e^x + x - 3 = 0$, giving your answer correct to 4 decimal places.   [5]

                                                                             Total 7 marks

4. a) The first four terms of the binomial expansion of $(1 + 2x)^8$ in ascending powers of x are
    $1 + ax + bx^2 + cx^3$. Find the values of the integers a, b and c. [4]

    b) Hence, find the coefficient of x^3 in the expansion of
    $$\left(1 + \frac{1}{2}x\right)(1 + 2x)^8$$
    [3]

    Total 7 marks

5. a) Find the binomial expansion of
    $$\sqrt{(1 - 8x)}, |x| < \frac{1}{8},$$
    in ascending powers of x up to and including the term in x^3, simplifying each term. [4]

    b) Show that, when $x = \frac{1}{100}$, the **EXACT** value of $\sqrt{(1 - 8x)}$ is $\frac{\sqrt{23}}{5}$. [2]

    c) Hence, using the expansion in part a), obtain an approximation to $\sqrt{23}$, giving your answer to 5 decimal places. [3]

    Total 9 marks

6. a) Obtain the first four non-zero terms in the Maclaurin series expansion of $f(x) = \cos 2x$. [2]

    b) Hence, using the double angle formula for $\cos 2x$, find the first three non-zero terms in the expansion of $sin^2 x$. [3]

    c) Use the result from b) to show that
    $$\lim_{x \to 0} \frac{sin^2 x - x^2}{x^4} = \frac{-1}{3}$$
    [2]

    Total 7 marks

7. a) Verify the identity
    $$\frac{2r - 1}{r(r - 1)} - \frac{2r + 1}{r(r + 1)} \equiv \frac{2}{(r - 1)(r + 1)}.$$
    [4]

    b) Hence, using the method of summation by differences, prove that
    $$\sum_{r=2}^{n} \frac{2}{(r - 1)(r + 1)} = \frac{3}{2} - \frac{2n + 1}{n(n + 1)}$$
    [4]

    Total 8 marks

8. Prove by mathematical induction that

$$\sum_{r=1}^{n} r(r!) = (n+1)! - 1 \ for \ all \ n \in \mathbb{N}$$

[8]

Total 8 marks

END OF TEST