HARRISON COLLEGE INTERNAL EXAMINATION, APRIL 2018

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 1 – TEST 3 (PREVIEW)

Time: 1 Hour & 20 minutes

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1. (a) Determine

   (i) $\lim_{x\to 3}\dfrac{x^3-9x}{x-3}$ [18]

   (ii) $\lim_{x\to 0}\dfrac{\sin 5x}{2x}$ $\left[\frac{5}{2}\right]$

   (b) Find the values of x for which $\frac{x^2+1}{|2x+3|-6}$ is discontinuous. $\left[-\frac{9}{2},\frac{3}{2}\right]$

   (c) A function $f(x)$ is defined as

$$f(x)=\begin{cases}x+2 & x\leq 2\\ x^2 & x>2\end{cases}$$

   (i) Find $\lim_{x\to 2} f(x)$. [4]

   (ii) Determine whether $f(x)$ is continuous at $x=2$. Give a reason for your answer. [Yes]

   (d) Differentiate $f(x)=\sin 2x$ using first principles. $[2\cos 2x]$

Total 23 Marks

2. Given that $y=8x+\frac{1}{x}$, determine the equation of the tangent to the curve at the point where $x=1$.

$[y=7x+2]$

TOTAL 6 Marks

3. The curve C has equation $y=\frac{x}{1+x^2}$.

   (i) Show that $\frac{dy}{dx}=\frac{1-x^2}{(1+x^2)^2}$

   (ii) Determine the coordinates of the stationary points on C. $\left[\left(1,\frac{1}{2}\right),\left(-1,-\frac{1}{2}\right)\right]$

Total 7 Marks

4.

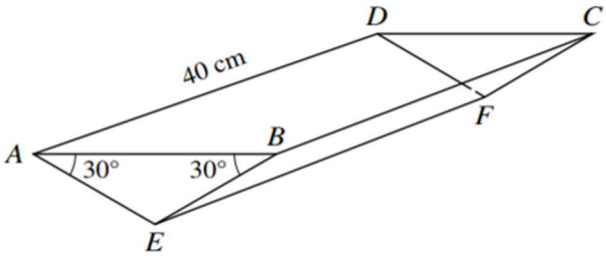


Fig. 1

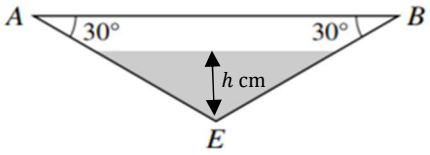


Fig. 2

Fig. 1 shows an open tank in the shape of a triangular prism. The vertical ends ABE and DCF are identical isosceles triangles, angle ABE = angle $BAE = 30°$. The length of AD is 40 cm. The tank is fixed in position with the open top $ABCD$ horizontal. Water is poured into the tank at a constant rate of 100 cm^3s^{-1}. The depth of water, t seconds after filling starts, is h cm (see Fig. 2).

(i) Show that, when the depth of water in the tank is h cm, the volume, V cm^3, of water in the tank is given by $V = \left(40\sqrt{3}\right)h^2$.

(ii) Find the rate at which h is increasing when $h = 4$. $\left[\frac{5\sqrt{3}}{16}\right]$

Total 6 Marks

5. The parametric equations of a curve are given by

$$x = \cos\theta\,, y = \sin 2\theta\,, \qquad 0 \le \theta \le 2\pi$$

find $\frac{dy}{dx}$. $\left[-\frac{2\cos 2}{\sin\theta}\right]$

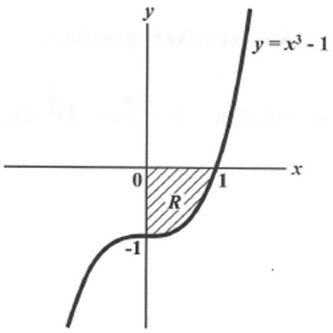
Total 4 Marks

6. Use the substitution $u = \sin x + 2$ to show that

$$\int \cos x\,(2 + \sin x)^6\; dx = \frac{(2 + \sin x)^7}{7} + c$$

Total 8 Marks

7. The diagram below represents the finite region R which is enclosed by the curve $y = x^3 - 1$ and the lines $x = 0$ and $y = 0$.



Calculate the volume of the solid that results from rotating R about the $y-$ axis. $\left[\frac{3\pi}{5}\right]$

Total 6 Marks