Name of Institution: Harrison College
School Code: 030014
CAPE Pure Mathematics 2018
UNIT 1 TEST 1
DATE: 20 March 2018

1) $\sim p \wedge (\sim q \vee p)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge p)$ 1

$= (\sim p \wedge \sim q) \vee 0$ 1

$= (\sim p \wedge \sim q)$

$= \sim (p \vee q)$ 1 CAO

2) (a) (i) $\sum_{r=1}^{n} (2r)^3$

$= 8 \sum_{r=1}^{n} r^3$ 1

$= 8 \times \frac{n^2(n+1)^2}{4}$ 1

$= 2n^2(n+1)^2$

$= 2n^4 + 4n^3 + 2n^2$ 1 FT

(ii) $\sum_{r=1}^{n} (2r)^3 = 72$

$2n^2(n+1)^2 = 72$

$n^2(n+1)^2 = 36$

$[n(n+1)]^2 = 36$

$n(n+1) = \pm 6$ 1

$n(n+1) = 6$, $n(n+1) = -6$

$n^2+n-6=0$ $n^2+n+6=0$

$(n+3)(n-2)=0$ 1 No Real n

$n=-3$, $n=2$ 1

Take $n=2$ 1

2) (a) (ii) Alternatively

$2n^4 + 4n^3 + 2n^2 = 72$

$n^4 + 2n^3 + n^2 - 36 = 0$

$$
\begin{array}{r|l}
 & n^3 - n^2 + 4n - 12 \\
\hline
n+3 & n^4 + 2n^3 + n^2 - 36 \\
 & \underline{n^4 + 3n^3} \\
 & \quad -n^3 + n^2 \\
 & \quad \underline{-n^3 - 3n^2} \\
 & \qquad 4n^2 + 0n \\
 & \qquad \underline{4n^2 + 12n} \\
 & \qquad\quad -12n - 36 \\
 & \qquad\quad \underline{-12n - 36} \\
 & \qquad\qquad 0 \quad 0
\end{array}
$$

1

$$
\begin{array}{r|l}
 & n^2 + n + 6 \\
\hline
n-2 & n^3 - n^2 + 4n - 12 \\
 & \underline{n^3 - 2n^2} \\
 & \quad n^2 + 4n \\
 & \quad \underline{n^2 - 2n} \\
 & \qquad 6n - 12 \\
 & \qquad \underline{6n - 12} \\
 & \qquad\quad 0 \quad 0
\end{array}
$$

1

$(n+3)(n-2)(n^2+n+6) = 0$

$n = -3, \quad n = 2$ 1

Take $n = 2$ 1

2) (b) $\dfrac{3\sqrt{5}-4}{2\sqrt{5}+1}$

$= \dfrac{(3\sqrt{5}-4)(2\sqrt{5}-1)}{(2\sqrt{5}+1)(2\sqrt{5}-1)}$ (1)

$= \dfrac{6\times 5 - 3\sqrt{5} - 8\sqrt{5} + 4}{(2\sqrt{5})^2 - (1)^2}$

$\left.\begin{aligned} &= \dfrac{34 - 11\sqrt{5}}{4\times 5 - 1} \\ &= \dfrac{34 - 11\sqrt{5}}{19} \end{aligned}\right\}$ (1)

$= \dfrac{34}{19} - \dfrac{11\sqrt{5}}{19}$ (1) + (1) FT

3) (i) Let $f(x) = 2x^3 + ax^2 + bx + 6$

$f(2) = 0$

i.e. $2(2)^3 + a(2)^2 + b(2) + 6 = 0$

$16 + 4a + 2b + 6 = 0$

$2a + b = -11$ eq$^=$ (1) (1).

$f(-1) = -12$

i.e. $2(-1)^3 + a(-1)^2 + b(-1) + 6 = -12$

$-2 + a - b + 6 = -12$

$a - b = -16$ eq$^=$ (2) (1)

$3a = -27$

$a = -9$ sub. into (2) (1) FT

$-9 - b = -16$

$7 = b$ (1) FT

3) ii) $2x^3 + ax^2 + bx + 6 = 0$

$2x^3 - 9x^2 + 7x + 6 = 0$

$$\begin{array}{r|l} & 2x^2 - 5x - 3 \\ \hline x-2 & 2x^3 - 9x^2 + 7x + 6 \\ & \underline{2x^3 - 4x^2} \\ & \quad -5x^2 + 7x \\ & \quad \underline{-5x^2 + 10x} \\ & \qquad -3x + 6 \\ & \qquad \underline{-3x + 6} \\ & \qquad 0 \quad 0 \end{array}$$ 1

$(x-2)(2x^2 - 5x - 3) = 0$

$(x-2)(x-3)(2x+1) = 0$

$x = 2,\ x = 3,\ x = -\frac{1}{2}$ 1 + 1 + 1

Give 1 mark for Factors only.

4) Let P_n be the proposition $\sum_{r=1}^{n} \frac{1}{(3r-2)(3r+1)} = \frac{n}{3n+1}\ \forall n \in \mathbb{N}$

BASIC STEP

$r=1$: LHS $= \frac{1}{(1)(4)} = \frac{1}{4}$ 1

$n=1$: RHS $= \frac{1}{3+1} = \frac{1}{4}$ 1

$\therefore P_1$ is TRUE

INDUCTIVE STEP

Assume P_k is true i.e. $\sum_{r=1}^{k} \frac{1}{(3r-2)(3r+1)} = \frac{k}{3k+1}\ \forall k \in \mathbb{N}, k > 1$ 1

We are required to show that $P_{k+1} = \frac{(k+1)}{3(k+1)+1}$

$= \frac{(k+1)}{3k+4}$

Now $P_{k+1} = P_k + (k+1)\text{st term}$

$$= \frac{k}{3k+1} + \frac{1}{[3(k+1)-2][3(k+1)+1]}$$ ①

$$= \frac{k}{3k+1} + \frac{1}{[3k+1][3k+4]}$$

$$= \frac{k[3k+4]+1}{[3k+1][3k+4]}$$

$$= \frac{3k^2+4k+1}{[3k+1][3k+4]}$$

$$= \frac{(\cancel{3k+1})(k+1)}{[\cancel{3k+1}][3k+4]}$$

$$= \frac{(k+1)}{[3k+4]} \quad \text{AS REQ'D}$$ ①

CONCLUSION

$P_k \Rightarrow P_{k+1}$

i.e. $P_1 \Rightarrow P_2$, $P_2 \Rightarrow P_3$ etc

Hence, By MI P_n is true $\forall\, n \in \mathbb{N}$ ①

5) (a) $3\log_8 x - 5 = 2\log_x 8$

$3\log_8 x - 5 = 2 \times \frac{1}{\log_8 x}$ ①

$3(\log_8 x)^2 - 5\log_8 x = 2$

$3(\log_8 x)^2 - 5\log_8 x - 2 = 0$

$(3\log_8 x + 1)(\log_8 x - 2) = 0$ ①

$\log_8 x = \frac{-1}{3}$, $\log_8 x = 2$

$x = 8^{-1/3}$, $x = 8^2$

$x = \frac{1}{2}$, $x = 64$ ① + ①

5) (b) $e^{2x} + 2e^{-2x} = 3$

$e^{2x} + \frac{2}{e^{2x}} - 3 = 0$

$(e^{2x})^2 + 2 - 3e^{2x} = 0$

$(e^{2x})^2 - 3e^{2x} + 2 = 0$ (1)

$(e^{2x} - 2)(e^{2x} - 1) = 0$ (1)

$e^{2x} = 2, \quad e^{2x} = 1$

$2x = \ln 2, \quad 2x = \ln 1$

$x = \frac{1}{2}\ln 2, \quad x = 0$ (1) + (1)

6) (i) $N \propto \left(\frac{3}{2}\right)^t$

$N = k \cdot \left(\frac{3}{2}\right)^t$

At $t = 0, \quad N = 3200$

$3200 = k \cdot \left(\frac{3}{2}\right)^0$

$3200 = k$ (1) SOI

$\therefore N = 3200 \times \left(\frac{3}{2}\right)^t$

when $t = 3, \quad N = 3200 \times \left(\frac{3}{2}\right)^3$

$= 10\ 800$ (1)

(ii) $30\ 000 = 3200 \left(\frac{3}{2}\right)^t$

$\frac{30\ 000}{3200} = \left(\frac{3}{2}\right)^t$ (1)

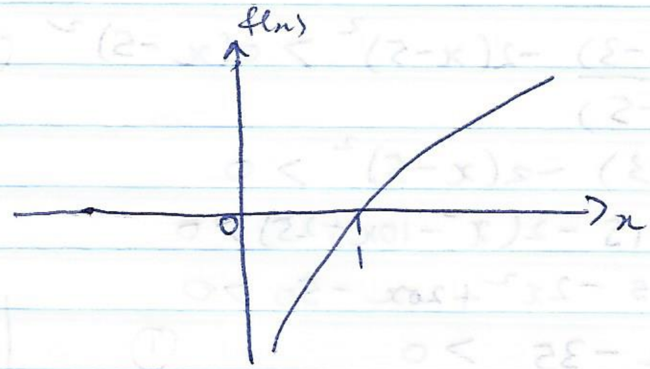
$\frac{75}{8} = \left(\frac{3}{2}\right)^t$

$\frac{\ln\left(\frac{75}{8}\right)}{\ln\left(\frac{3}{2}\right)} = t$ (1) (1)

5.52 days $= t$ (1)

7) $f: x \longrightarrow \frac{1}{2}\ln x$

(i)



Orientation ①

(1,0) Seen ①

(ii) Let $y = f(x)$

i.e. $y = \frac{1}{2}\ln x$

$x = \frac{1}{2}\ln y$ ①

$2x = \ln y$

$e^{2x} = y$

i.e. $e^{2x} = f^{-1}(x)$ ①

(iii) $g: x \longrightarrow e^{x} + 2$

$gf(x) = g\left(\frac{1}{2}\ln x\right)$

$= e^{\frac{1}{2}\ln x} + 2$ ①

$= e^{\ln x^{\frac{1}{2}}} + 2$ ①

$\left.\begin{aligned} &= x^{\frac{1}{2}} + 2 \\ &= \sqrt{x} + 2 \end{aligned}\right\}$ ①

8) $\frac{x-3}{x-5} - 2 > 0$

$(x-5)^2 \cdot \frac{(x-3)}{(x-5)} - 2(x-5)^2 > 0(x-5)^2$ 1

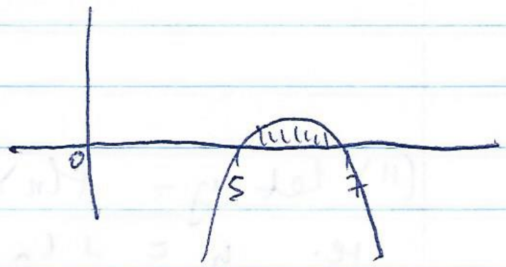
$(x-5)(x-3) - 2(x-5)^2 > 0$

$x^2 - 8x + 15 - 2(x^2 - 10x + 25) > 0$

$x^2 - 8x + 15 - 2x^2 + 20x - 50 > 0$

$-x^2 + 12x - 35 > 0$ 1

$(-x+5)(x-7) > 0$ 1



Consider $(-x+5)(x-7) = 0$

$x = 5, \quad x = 7$

solution $5 < x < 7$ 1 + 1

9) $|2x-5| - 7 \geq -4$

$|2x-5| \geq 3$ 1

$+(2x-5) \geq 3$ 1 1

or $-(2x-5) \geq 3$ 2 1

From 1, $x \geq 4$ 1

From 2, $2x \leq 2$ 1

$x \leq 1$ 1

Alternatively

$|2x-5| - 7 \geq -4$

$+\sqrt{(2x-5)^2} - 7 \geq -4$ 1

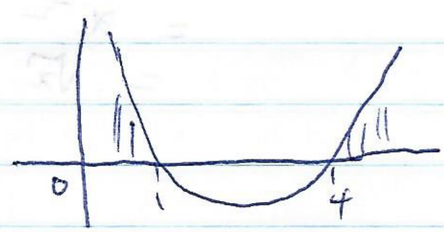
$+\sqrt{(2x-5)^2} \geq 3$ 1

$(2x-5)^2 \geq 3^2$ 1

$4x^2 - 20x + 25 - 9 \geq 0$

$x^2 - 5x + 4 \geq 0$

$(x-4)(x-1) \geq 0$ 1



$x \leq 1, \quad x \geq 4$ 1 + 1