PURE MATHEMATICS
UNIT 1 - TEST 3 (2017 PREVIEW)

TIME: 1 Hour & 20 minutes

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1. (a) Find $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$ (ans: $\frac{1}{6}$)

(b) Given

$$f(x) = \begin{cases} x^2 + 1, & x > 3 \\ 4 + px, & x \leq 3 \end{cases}$$

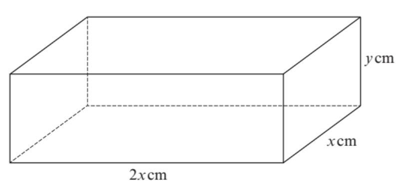
Find the values of p if $f(x)$ is continuous at x =3. (ans: p = 2)

(c) Prove by using first principles that if $y = \sin 2x$ then $\frac{dy}{dx} = 2\cos 2x$.

2. (a) (i) Find $\frac{dy}{dx}$ when $y = \frac{3x+2}{5x-1}$ (ans: $\frac{-13}{(5x+1)^2}$)

(ii) Find $\frac{dy}{dx}$ when $y = x\sqrt[3]{1-x}$ (ans: $-\frac{1}{3}x(1-x)^{\frac{-2}{3}} + \sqrt[3]{1-x}$)

(b) The diagram shows a **closed** box in the form of a cuboid. The length of the box is 2x cm, its width is x cm and its height is y cm.



The total surface area of the box is 108 cm^3.

(i) Write down an equation involving x and y and show that

$$xy = 18 - \frac{2}{3}x^2$$

(ii) Hence show that the volume V cm^3 of the box is given by

$$V = 36x - \frac{4}{3}x^3$$

(iii) Find the maximum value of V, showing that the value you have found is maximum value. (ans: V_{max} = 72cm^3)

(c) A curve is given parametrically by

$$x = \frac{1}{3}sin4\theta + 1 \qquad y = 1 - cos4\theta$$

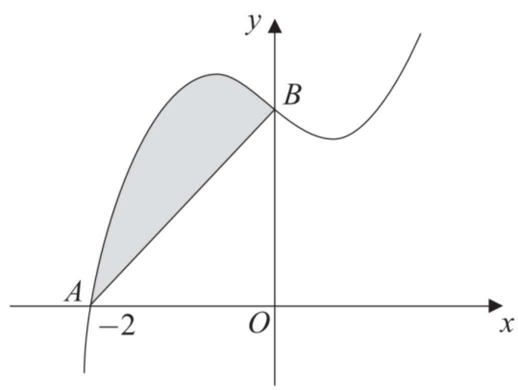
(i) Find $\frac{dy}{dx}$ in terms of θ (ans: $3tan4\theta$)

(ii) Find the equation of the normal to the given curve at the point where $\theta = \frac{\pi}{16}$.

(ans: $3y + x = 4 - \frac{4\sqrt{2}}{3}$)

3. (a)

The curve with equation $y = x^3 - x + 6$ is sketched below.



The curve cuts the x-axis at the point $A\,(-2,\ 0)$ and the y-axis at the point B.

(i) State the y-coordinate of the point B.

(ii) Find $\int_{-2}^{0} (x^3 - x + 6)\,dx$.

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - x + 6$ and the line AB.

(ans: (i) (0, 6) (ii) 10 (iii) 4)

(b) (i) Find $\int_0^5 \sin(\frac{x}{3})\ dx$. (ans: 2.71)

(ii) Using the substitution $u = 2x - 1$, find

$$\int_1^5 x\sqrt{2x-1}\,dx$$ (ans: $\frac{1182}{60}$)

(c) The gradient of a curve is given by $\frac{dy}{dx} = (2x-1)^4$.

The point $(1,\ 2)$ lies on the curve. Find the equation of the curve.

(ans: $y = \frac{(2x-1)^5}{10} + \frac{19}{10}$)