HARRISON COLLEGE INTERNAL EXAMINATION, APRIL 2017
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS
UNIT 1 - TEST 3

TIME: 1 Hour & 20 minutes

This examination paper consists of 3 printed pages.
The paper consists of 3 questions.
The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Number your questions carefully and do **NOT** write your solutions to different questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Electronic calculator (non-programmable, non-graphical)

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1. (a) Find $\lim_{x \to 7} \frac{x-7}{\sqrt{x+2}-3}$ [4]

(b) The function f on $\mathbb{R}$ is defined by

$$f(x) = \begin{cases} 2x-1, & x \leq 2 \\ ax^2+bx-5, & 2 < x < 3 \end{cases}$$

where $a, b \in \mathbb{R}$

Given that f and its derivative f' are continuous for all values in the domain of f, find the values of a and b. [6]

(c) Prove by using first principles that if $y = \cos x$ then $\frac{dy}{dx} = -\sin x$. [5]

TOTAL 15 marks

2. (a) (i) Find $\frac{dy}{dx}$ when $y = \frac{2x-1}{4x+3}$ [3]

(ii) The strength of a person's reaction, R, to a certain drug is given by

$$R(Q) = Q(65 - \frac{Q}{3})^{\frac{1}{2}}$$

Where Q represents the quantity of the drug given to the patient.

The sensitivity to the drug is given by $R'(Q)$. Find the sensitivity to the drug if the patient is given 87 units of the drug. [4]

(b) A cylindrical box will be tied up with ribbon as shown in the figure below.

The longest piece of ribbon available is 130 cm long and 10 cm of that is required for the bow. Given that r and h are the radius and the height of the box respectively

(i) Express h in terms of r. [2]

(ii) Show that the volume of the box is given by $V = 30\pi r^2 - 2\pi r^3$. [2]

(iii) Hence determine the radius and height of the box with the largest possible volume.

[4]

(c) A curve is defined by the parametric equations

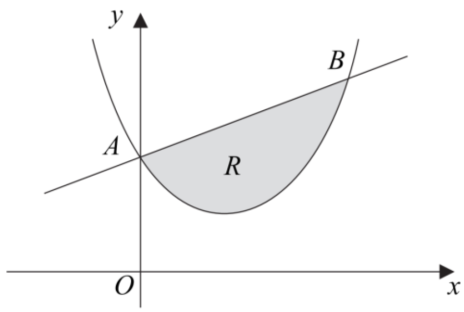
$$x = 2\theta - \sin 2\theta \qquad y = 2 - \cos 2\theta$$

(i) Find $\frac{dy}{dx}$ in terms of θ. [3]

(ii) Find the equation of the tangent to the given curve at the point where $\theta = \frac{\pi}{4}$. [4]

TOTAL 22 marks

3. (a) The curve C with equation $y = x^2 - 3x + 5$ and the straight line $y = x + 5$ intersect at the point $A(0, 5)$ and at the point B as shown in the diagram below.



(i) Find the coordinates of the point B. [3]

(ii) Find $\int (x^2 - 3x + 5)\, dx$ [3]

(iii) Find the area of the shaded region R bounded by the curve C and the line segment AB. [4]

(b) (i) Find $\int_0^{\frac{\pi}{2}} \cos(2x) + 1 \;dx$. [3]

(ii) Using the substitution $u = 1 - x$, find

$$\int_0^1 x\sqrt{1-x}\, dx$$ [6]

(c) The gradient of a curve is given by $\frac{dy}{dx} = \frac{1}{\sqrt{2x}}$. The point $(2, 5)$ lies on the curve. Find the equation of the curve. [4]

TOTAL 23 marks

End of test

HARRISON COLLEGE INTERNAL EXAMINATIONS 2017: CAPE PURE MATHEMATICS [UNIT1 – TEST **3**]

SOLUTIONS AND MARK SCHEME

Question	Working	Marks & comments
1.(a)	$\lim_{x\to7}\frac{x-7}{\sqrt{x+2}-3}$	
	= $\lim_{x\to7}\frac{x-7}{\sqrt{x+2}-3}\times\frac{\sqrt{x+2}+3}{\sqrt{x+2}+3}$	1
	= $\lim_{x\to0}\frac{(x-7)\,(\sqrt{x+2}+3)}{x-7}$	1
	= $\lim_{x\to7}\sqrt{x+2}+3$	1
	= $\sqrt{9}+3$= 6	1 Total = 4 marks
(b)	$\lim_{x\to2^+}f(x)=\lim_{x\to2^-}f(x)=f(2)=2(2)-1=3$	1 seen or implied
	$4a+2b-5=3$	1
	$4a+2b=8$	
	$\lim_{x\to2^+}f^{\vert}(x)=\lim_{x\to2^-}f^{\vert}(x)=f^{\vert}(2)$	1 seen or implied
	$4a+b=2$	1
	Solving simultaneously	
	$4a+2b=8$ and $4a+b=2$	
	$a=-1$	1
	$b=6$	1 Total = 6 marks
(c)	$\frac{dy}{dx}=\lim_{h\to0}[\frac{f(x+h)-\ f(x)}{h}]$ therefore If $y=\cos x$	
	$\frac{dy}{dx}=\lim_{h\to0}[\ \frac{\cos(x+h)-\cos x}{h}]$	1
	= $\lim_{h\to0}\frac{\cos x\cos h-\sin x\sin h-cosx}{h}$	1
	= $\lim_{h\to0}\frac{\cos x(\cos h-1)}{h}-\frac{sn\ x\sin h}{h}$	
	= $cosx\ \lim_{h\to0}\frac{\cos h-1}{h}-\sin x\ \lim_{h\to0}\frac{\sin h}{h}$	1
	= $\cos x\ (0)-\cdots$	1
	= $0-\sin x\ (1)=\ -\sin x$	1 Total = 5 marks

2(a)(i)	$y = \dfrac{2x-1}{4x+3}$ $\dfrac{dy}{dx} = \dfrac{(\quad)(\quad)-(\quad)(\quad)}{(4x+3)^2}$ $= \frac{(4x+3)(2)-(\quad)(\quad)}{(4x+3)^2}$ $= \frac{(4x+3)(2)-(2x-1)(4)}{(4x+3)^2} = \frac{10}{(4x+3)^2}$	 1 1 1 Total = 3 marks
(a) (ii)	$R(Q) = Q(65 - \frac{Q}{3})^{\frac{1}{2}}$ $R^{\prime}(Q) = Q \times \frac{1}{2} \times (65 - \frac{Q}{3})^{\frac{-1}{2}} \times \frac{-1}{3}$ $+$ $(65 - \frac{Q}{3})^{\frac{1}{2}} \cdot (1)$ Substituting Q = 87 into his $R^{\prime}(Q)$ $R^{\prime}(Q) = \frac{43}{12} = 3.58$	 1 1 1 1 correct answer only Total = 4 marks
(b)(i)	$4h + 8r + 10 = 130$ $4h + 8r = 120$ $4h = 120 - 8r$ $h = 30 - 2r$	1 1 Total = 2marks
(b) (ii)	$V = \pi r^2 h$ $V = \pi r^2(30 - 2r)$ $V = 30\pi r^2 - 2\pi r^3$	1 1 Total = 2 marks

(b) (iii)	$\dfrac{dV}{dr} = 60\pi r - 6\pi r^2$ $= 0$ $\therefore r = 10$ cm $h = 30 - 2(10) = 10$ cm	1 1 1 1 Total = 4 marks
(c) (i)	$x = 2\theta - \sin 2\theta \quad \therefore \frac{dx}{d\theta} = 2 - 2\cos 2\theta$ $y = 2 - 2\cos 2\theta \quad \therefore \frac{dy}{d\theta} = 2\sin 2\theta$ $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{2\sin 2\theta}{2 - 2\cos 2\theta}$	1 1 1(give mark for his $\frac{dy}{d\theta} \div \frac{dx}{d\theta}$). Total = 3 marks

3 (c) (ii)	For $\theta = \frac{\pi}{4}$ $\frac{dy}{dx} = \frac{2\sin\frac{\pi}{2}}{2-2\cos\frac{\pi}{2}} = 1$ $x = 2\left(\frac{\pi}{4}\right) - \sin\frac{\pi}{2} = \frac{\pi}{2} - 1$ $y = 2 - \cos\frac{\pi}{2} = 2$ Equation of tangent: $y - 2 = 1(x - \left(\frac{\pi}{2} - 1\right))$	 1 (using his $\frac{dy}{dx}$) 1 1 1 Total = 4 marks

3(a) (i)	$x^2 - 3x + 5 = x + 5$ $x^2 - 4x = 0$ $x(x-4) = 0 \quad x = 0 \quad x = 4$ Coordinates of B (4, 9)	 1 1 1 Total = 3 marks
(a)(ii)	$\int x^2 - 3x + 5\ dx$ = $= \frac{x^3}{3}$ ... $-3\frac{x^2}{2}$ $+5x + constant$	 1 1 1 Total = 3 marks
(a)(iii)	Shaded region R = area under line AB = $\frac{1}{2}(4)(9+5) = 28$ minus area under curve AB= $\left[\frac{x^3}{3} - \frac{3x^2}{2} 3x + 5x\right]_0^4 = \frac{52}{3}$ area $= \frac{32}{3}$	 1 1 1 1 correct answer only Total = 4 marks
3(b) (i)	$\int_0^{\frac{\pi}{2}} \cos 2x + 1\, dx$ = $\frac{\sin 2x}{2} + x \ \Big\vert_0^{\frac{\pi}{2}}$ = $\left(\frac{\sin\pi}{2} + \frac{\pi}{2}\right) - \left(\frac{\sin 0}{2} + 0\right)$ = $\frac{\pi}{2}$	 1 1 for substituting limits and working in radian measure 1 correct answer only

3 (b) (ii)	$u = 1 - x \quad \frac{du}{dx} = -1 \qquad dx = -du$ when $x = 0 \;\; u = 1$ and $x = 1 \;\; u = 0$ substituting into given integral $\int_1^0 (1-u)u^{\frac{1}{2}}\ (-1)du = \int_0^1 u^{\frac{1}{2}} - u^{\frac{3}{2}}\ \ dx$ $= \frac{2u^{\frac{3}{2}}}{3} -$ $\frac{2u^{\frac{5}{2}}}{5}\Big]_0^1$ $= \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$	1 1 1 1 1 1 correct answer only Total = 6 marks
3 (c)	$\frac{dy}{dx} = \frac{1}{\sqrt{2x}} = (2x)^{\frac{-1}{2}}$ $y = \int (2x)^{\frac{-1}{2}}\, dx$ $y = (2x)^{\frac{1}{2}}$ + c Substituting x = 2 and y =5 to find "c" $5 = \sqrt{2 \times 2} + c \qquad$ c = 3 So equation of curve $y = \sqrt{2x} + 3$	 1 attempting to integrate 1 integrating correctly and show "c" 1 attempting to find value of "c" 1 Total = 4 marks