HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2020
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT I – TEST 2
1 hour and 20 minutes

NAME OF STUDENT: ____
SCHOOL CODE: 030014______
DATE: ______________________

This examination paper consists of 9 printed pages and 2 blank pages for extra working.This paper consists of **6** questions.The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly in the space above.
2. Answer **ALL** questions in the **SPACES PROVIDED**.
3. Number your questions carefully and **DO NOT write your solutions to different questions beside each other**.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Scientific calculator (non-programmable, non-graphical)

1. (a) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \qquad x \neq (n - \tfrac{1}{4})\pi,\ n \in \mathbb{Z},$

(2)

(ii) $\tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}.$

(3)

(b) Hence, or otherwise, show that the equation

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta+\sin\theta}\right)=\frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(c) Solve, for $0 \leqslant \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)

Total 12 Marks

2. (a) Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. (3)

(b) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$. (2)

(c) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places.

(5)

Total 10 Marks

3. A curve has parametric equations

$$x = 2\cot t, \quad y = 2\sin^2 t, \quad 0 < t \leqslant \frac{\pi}{2}.$$

Find a cartesian equation of the curve in the form $y = f(x)$. (4)

Total 4 Marks

4. The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(a) Find the values of a and b.

(3)

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(b) Find the position vector of point P.

(6)

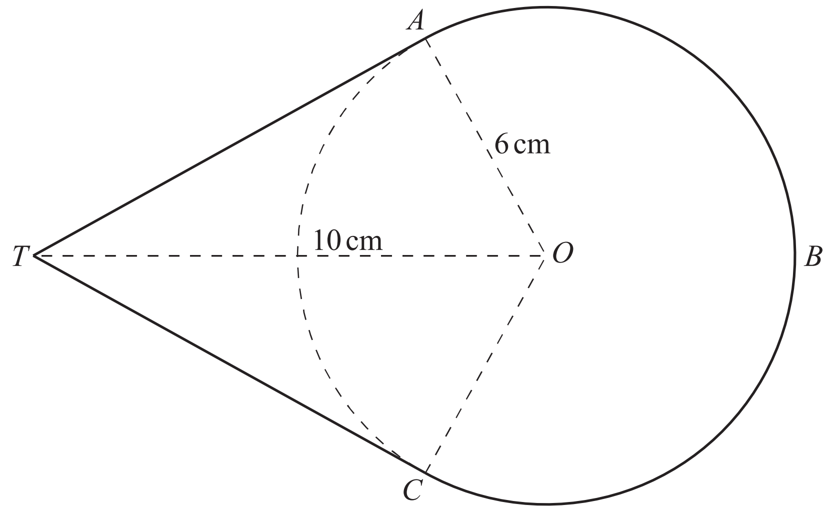
Given that B has coordinates (5, 15, 1),

(c) show that the points A, P and B are collinear and find the ratio $AP : PB$.

(4)

Total 13 Marks

5



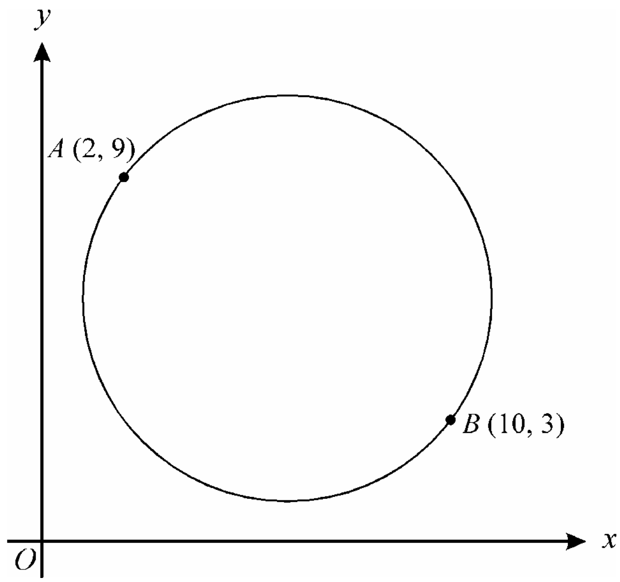
The points A, B and C lie on a circle centre O, radius 6 cm. The tangents to the circle at A and C meet at the point T. The length of OT is 10 cm. Find

(i) the angle TOA in radians, [2]

(ii) the area of the region $TABCT$, [6]

Total 8 Marks

6



The diagram shows a circle which passes through the points $A(2, 9)$ and $B(10, 3)$. AB is a diameter of the circle.

(i) Calculate the radius of the circle and the coordinates of the centre. [4]

(ii) Show that the equation of the circle may be written in the form $x^2 + y^2 - 12x - 12y + 47 = 0$. [3]

(iii) The tangent to the circle at the point *B* cuts the *x*-axis at *C*. Find the coordinates of *C* [6]

Total 13 Marks

END OF TEST

EXTRA SPACE

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Question No. [ ]

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No. ☐

Question Number	Scheme		
1.	(a) (i) Use of $\cos 2x = \cos^2 x - \sin^2 x$ in an attempt to prove the identity.	M1	
	$\dfrac{\cos 2x}{\cos x + \sin x} = \dfrac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x$ ∗ cso	A1	**(2)**
	(ii) Use of $\cos 2x = 2\cos^2 x - 1$ in an attempt to prove the identity.	M1	
	Use of $\sin 2x = 2\sin x \cos x$ in an attempt to prove the identity.	M1	
	$\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x) = \cos^2 x - \cos x \sin x - \frac{1}{2}$ ∗ cso	A1	**(3)**
	(b) $\cos\theta(\cos\theta - \sin\theta) = \frac{1}{2}$ Using (a)(i)	M1	
	$\cos^2\theta - \cos\theta\sin\theta - \frac{1}{2} = 0$		
	$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ Using (a)(ii)	M1	
	$\cos 2\theta = \sin 2\theta$ ∗	A1	**(3)**
	(c) $\tan 2\theta = 1$		
	$2\theta = \frac{\pi}{4}, \left(\frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}\right)$ any one correct value of 2θ	A1	
	$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ Obtaining at least 2 solutions in range	M1	
	The 4 correct solutions	A1	**(4)**
	If decimals $(0.393, 1.963, 3.534, 5.105)$ or degrees $(22.5°, 112.5°, 202.5°, 292.5°)$ are given, but all 4 solutions are found, penalise one A mark only. Ignore solutions out of range.		**[12]**

2.	(*a*)	Complete method for *R*: e.g. $R\cos\alpha = 3$, $R\sin\alpha = 2$, $R = \sqrt{(3^2 + 2^2)}$	M1
		$R = \sqrt{13}$ or 3.61 (or more accurate)	A1
		Complete method for $\tan\alpha = \frac{2}{3}$ [Allow $\tan\alpha = \frac{3}{2}$]	M1
		$\alpha = 0.588$ (Allow 33.7°)	A1 (4)
	(*b*)	Greatest value = $\left(\sqrt{13}\right)^4 = 169$	M1, A1 (2)
	(*c*)	$\sin(x + 0.588) = \frac{1}{\sqrt{13}}$ (= 0.27735…) $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$	M1
		$(x + 0.588) = 0.281(03\ldots$ or 16.1°	A1
		$(x + 0.588) = \pi - 0.28103\ldots$ Must be $\pi -$ their 0.281 or 180° – their 16.1°	M1
		or $(x + 0.588) = 2\pi + 0.28103\ldots$ Must be $2\pi +$ their 0.281 or 360° + their 16.1°	M1
		$x = 2.273$ or $x = 5.976$ (awrt) Both (radians only)	A1 (5)
		If 0.281 or 16.1° not seen, correct answers imply this A mark	**(11 marks)**
3.		Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$, or equivalent, to eliminate *t*	M1
		$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$ correctly eliminates *t*	A1
		$y = \frac{8}{4 + x^2}$ cao	A1
		The domain is $x \ldots 0$	B1 **(4)** **[12]**
An alternative in (c)			
		$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}$; $\cos t = \frac{x}{2}\sin t = \frac{x}{2}\left(\frac{y}{2}\right)^{\frac{1}{2}}$	
		$\sin^2 t + \cos^2 t = 1 \Rightarrow \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$	M1 A1
		Leading to $y = \frac{8}{4 + x^2}$	A1

Question Number	Scheme		Marks
4. (a)	Equating **i** ; $0 = 6 + \lambda \Rightarrow \lambda = -6$	$\underline{\lambda = -6}$ Can be implied	B1 $\Rightarrow$ d
	Using $\lambda = -6$ and		
	equating **j** ; $a = 19 + 4(-6) = -5$	For inserting **their stated** λ into either a correct **j** or **k** component Can be implied.	M1 $\Rightarrow$ d
	equating **k** ; $b = -1 - 2(-6) = 11$	$a = -5$ and $b = 11$	A1
			[3]
	With no working… … only one of a or b stated correctly gains the first 2 marks. … both a and b stated correctly gains 3 marks.		
(b)	$\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$		
	direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$		
	$\overrightarrow{OP} \perp l_1 \Rightarrow \underline{\overrightarrow{OP} \bullet \mathbf{d} = 0}$	Allow <u>this statement</u> for M1 if $\overrightarrow{OP}$ and **d** are defined as above.	
	ie. $\underline{\begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 0}$ $\left(\text{or } \underline{x + 4y - 2z = 0}\right)$	Allow either of these two <u>underlined statements</u>	M1
	$\therefore\ 6 + \lambda + 4(19 + 4\lambda) - 2(-1 - 2\lambda) = 0$	Correct equation	A1 oe
	$6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0$	Attempt to solve the equation in λ	dM1
	$21\lambda + 84 = 0 \Rightarrow \lambda = -4$	$\lambda = -4$	A1
	$\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$	Substitutes their λ into an expression for $\overrightarrow{OP}$	M1
	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7)	A1
			[6]

Note: A similar method may be used by using $\overrightarrow{OP} = (0+\lambda)\mathbf{i} + (-5+4\lambda)\mathbf{j} + (11-2\lambda)\mathbf{k}$ and $\mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$

$\overrightarrow{OP} \bullet \mathbf{d} = 0$ yields $6 + \lambda + 4(-5 + 4\lambda) - 2(11 - 2\lambda) = 0$

This simplifies to $21\lambda - 42 = 0 \Rightarrow \lambda = 2$.

$\overrightarrow{OP} = (0+2)\mathbf{i} + (-5+4(2))\mathbf{j} + (11-2(2))\mathbf{k}$

$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$

Question Number	Scheme		Marks
Aliter (b) **Way 2**	$\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$		
	$\overrightarrow{AP} = (6+\lambda-0)\mathbf{i} + (19+4\lambda+5)\mathbf{j} + (-1-2\lambda-11)\mathbf{k}$		
	direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$		
	$\overrightarrow{AP} \perp \overrightarrow{OP} \Rightarrow \underline{\overrightarrow{AP} \bullet \overrightarrow{OP} = 0}$	Allow <u>this statement</u> for M1 if $\overrightarrow{AP}$ and $\overrightarrow{OP}$ are defined as above.	
	ie. $\underline{\begin{pmatrix} 6+\lambda \\ 24+4\lambda \\ -12-2\lambda \end{pmatrix} \bullet \begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} = 0}$	<u>underlined statement</u>	M1
	$\therefore (6+\lambda)(6+\lambda) + (24+4\lambda)(19+4\lambda) + (-12-2\lambda)(-1-2\lambda) = 0$	Correct equation	A1 oe
	$36 + 12\lambda + \lambda^2 + 456 + 96\lambda + 76\lambda + 16\lambda^2 + 12 + 24\lambda + 2\lambda + 4\lambda^2 = 0$	Attempt to solve the equation in λ	dM1
	$21\lambda^2 + 210\lambda + 504 = 0$		
	$\lambda^2 + 10\lambda + 24 = 0 \Rightarrow (\lambda = -6)\ \underline{\lambda = -4}$	$\lambda = -4$	A1
	$\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$	Substitutes their λ into an expression for $\overrightarrow{OP}$	M1
	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7)	A1
			[6]

Note: A similar method to way 2 may be used by using $\overrightarrow{OP} = (5+\lambda)\mathbf{i} + (15+4\lambda)\mathbf{j} + (1-2\lambda)\mathbf{k}$
and $\overrightarrow{AP} = (5+\lambda-0)\mathbf{i} + (15+4\lambda+5)\mathbf{j} + (1-2\lambda-11)\mathbf{k}$
$\overrightarrow{AP} \bullet \overrightarrow{OP} = 0$ yields $(5+\lambda)(5+\lambda) + (20+4\lambda)(15+4\lambda) + (-10-2\lambda)(1-2\lambda) = 0$
This simplifies to $21\lambda^2 + 168\lambda + 315 = 0$. $\lambda^2 + 8\lambda + 15 = 0 \Rightarrow (\lambda = -5)\ \underline{\lambda = -3}$
$\overrightarrow{OP} = (5-3)\mathbf{i} + (15+4(-3))\mathbf{j} + (1-2(-3))\mathbf{k}$
$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$

5 **(i)**	$\cos TOA = \frac{6}{10} \rightarrow$ $TOA = 0.927$	**M1** **A1**	any method
(ii)	area of major sector = $\frac{1}{2}6^2\left(2\pi - 2 \times their 0.927\right)$ (= 79.7)	**M2**	or **M1** for $\frac{1}{2}$ $6^2(2 \times$ *their* $0.927)$ **DM1** for $\pi \times 6^2 - \frac{1}{2}$ $6^2(2 \times$ *their* $0.927)$
	area of half kite = $\frac{1}{2}(6)\sqrt{10^2 - 6^2}$ (=24)	**M1**	
	area of kite × 2 (=48)	**DM1**	any method
	complete correct plan awrt 128	**DM1** **A1**	*their* major sector + *their* kite
(iii)	arc length = $6 \times (2\pi - 2 \times their 0.927) + 2 \times \sqrt{10^2 - 6^2}$) awrt 42.6	**M1** **A1**	complete correct method

6 **(i)**	$AB^2 = (10-2)^2 + (3-9)^2 = 100$	M1		For correct calculation method for AB^2
	Hence the radius is 5	A1		For correct value for radius
	Mid-point of AB is $\left(\frac{2+10}{2}, \frac{9+3}{2}\right)$	M1		For correct calculation method for mid-point
	Hence centre is $(6, 6)$	A1	**4**	For both coordinates correct
(ii)	Equation is $(x-6)^2 + (y-6)^2 = 5^2$	M1		For using correct basic form of circle equn
	This is $x^2 - 12x + 36 + y^2 - 12y + 36 = 25$	A1		For expanding at least one bracket correctly
	i.e. $x^2 + y^2 - 12x - 12y + 47 = 0$, as required	A1	**3**	For showing given answer correctly
(iii)	Gradient of AB is $\frac{3-9}{10-2} = -\frac{3}{4}$	M1		For finding the gradient of AB
		A1		For correct value $-\frac{3}{4}$ or equivalent
	Hence perpendicular gradient is $\frac{4}{3}$	A1√		For relevant perpendicular gradient
	Equation of tangent is $y - 3 = \frac{4}{3}(x - 10)$	M1		For using their perp grad and B correctly
	Hence C is the point $(\frac{31}{4}, 0)$	M1		For substituting $y = 0$ in their tangent eqn
		A1	**6**	For correct value $x = \frac{31}{4}$

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