

**HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2018**  
**CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**  
**SCHOOL BASED ASSESSMENT**  
**PURE MATHEMATICS**  
**UNIT 2 – TEST 2**  
**1 hour 20 minutes**

This examination paper consists of 3 pages.  
This paper consists of 6 questions.  
The maximum marks for this examination is 60.

**INSTRUCTIONS TO CANDIDATES**

1. Write in ink.
2. Write your name clearly on each sheet of paper used.
3. Answer **ALL** questions.
4. Do **NOT** do questions beside one another.
5. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

**EXAMINATION MATERIALS ALLOWED**

1. Mathematical formulae sheet
  2. Scientific Non-programmable calculator (non-graphical)
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1. (a) The sum of an infinite geometric sequence is 18 and the sum of the first three terms is  $12\frac{2}{3}$ . Find the first term. [5]
  
- (b) An arithmetic series, with first term 8 and common difference  $d$ , consists of 23 terms. Given that the sum of the last 3 terms is 5 times the sum of the first 3 terms.  
Find:
  - (i) the value of  $d$ . [3]
  - (ii) the sum of the first 15 terms. [2]

**Total 10 marks**
  
2. The sequence  $u_1, u_2, u_3 \dots$  is defined by  $u_1 = 2$  and  $u_{n+1} = 2u_n - 1$ .
  - (i) Find  $u_2$  and  $u_3$  and verify that  $\frac{1}{2}(u_4 - 1) = 4$  [3]
  - (ii) Hence suggest an expression in terms of  $n$  for  $u_n$ . [2]
  - (iii) Use induction to prove that your answer to part (ii) is correct. [5]

**Total 10 marks**

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3. (i) Show that  $\frac{1}{r} - \frac{1}{r+2} = \frac{2}{r(r+2)}$ . [2]

(ii) Hence, find an expression, in terms of  $n$ , for

$$\sum_{r=1}^n \frac{2}{r(r+2)}$$

[5]

(iii) Determine

(a) [1]

$$\sum_{r=1}^{\infty} \frac{2}{r(r+2)}$$

(b) [2]

$$\sum_{r=n+1}^{\infty} \frac{2}{r(r+2)}$$

**Total 10 marks**

4. The function  $f$  is defined by  $f(x) = \ln(1 + e^x)$

(a) Use Maclaurin's theorem to show that when  $f(x)$  is expanded in ascending powers of  $x$ :

(i) the first three terms are  $\ln 2 + \frac{1}{2}x + \frac{1}{8}x^2$ . [6]

(ii) the coefficient of  $x^3$  is zero. [3]

(b) Hence write down the first two non-zero terms in the expansion, in ascending powers of  $x$ , of  $\ln\left(\frac{1+e^x}{2}\right)$ . [1]

**Total 10 marks**

5. (a) The coefficient of  $x^2$  in the expansion of  $\left(x + \frac{1}{ax}\right)^8$  is 7.

Find the possible value of  $a$ . [3]

- (b) (i) Find the binomial expansion of  $(1 + 6x)^{\frac{2}{3}}$  up to and including the term in  $x^2$ . [2]

- (ii) Find the binomial expansion of  $(8 + 6x)^{\frac{2}{3}}$  up to and including the term in  $x^2$ . [3]

- (iii) Use your answer from (b) (ii) to find an estimate for  $\sqrt[3]{100}$  in the form  $\frac{a}{b}$  where

$a$  and  $b$  are integers. [2]

**Total 10 marks**

6. The equation

$$24x^3 + 36x^2 + 18x - 5 = 0$$

has one real root  $\alpha$ .

- (a) Show that  $\alpha$  lies in the interval  $0.1 < x < 0.2$ . [3]

- (b) Starting from the interval  $0.1 < x < 0.2$ , use interval bisection twice to obtain an interval of width 0.025 within which  $\alpha$  must lie. [3]

- (c) Taking  $x_1 = 0.2$  as a first approximation of  $\alpha$ , use the Newton-Raphson method to find a second approximation  $x_2$  of  $\alpha$ . Give your answer to 4 decimal places. [4]

**Total 10 marks**