HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2018 CARIBBEAN ADVANCED PROFICIENCY EXAMINATION SCHOOL BASED ASSESSMENT PURE MATHEMATICS UNIT 2 – TEST 2 1 hour 20 minutes

This examination paper consists of 3 pages. This paper consists of 6 questions. The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

- 1. Write in ink.
- 2. Write your name clearly on each sheet of paper used.
- 3. Answer ALL questions.
- 4. Do **NOT** do questions beside one another.
- 5. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

- 1. Mathematical formulae sheet
- 2. Scientific Non-programmable calculator (non-graphical)
- 1. (a) The sum of an infinite geometric sequence is 18 and the sum of the first three terms is $12\frac{2}{3}$. Find the first term. [5]
 - (b) An arithmetic series, with first term 8 and common difference *d*, consists of 23 terms. Given that the sum of the last 3 terms is 5 times the sum of the first 3 terms. Find:
 - (i) the value of d. [3]
 - (ii) the sum of the first 15 terms. [2] Total 10 marks
- 2. The sequence $u_1, u_2, u_3 \dots$ is defined by $u_1 = 2$ and $u_{n+1} = 2u_n 1$.

(i) Find u_2 and u_3 and verify that $\frac{1}{2}(u_4 - 1) = 4$ [3]	[3]
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- (ii) Hence suggest an expression in terms of n for u_n . [2]
- (iii) Use induction to prove that your answer to part (ii) is correct. [5]

Total 10 marks

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3. (i) Show that
$$\frac{1}{r} - \frac{1}{r+2} = \frac{2}{r(r+2)}$$
. [2]

(ii) Hence, find an expression, in terms of n, for

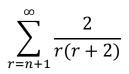
$$\sum_{r=1}^{n} \frac{2}{r(r+2)}$$
[5]

(iii) Determine

(a)

$$\sum_{r=1}^{\infty} \frac{2}{r(r+2)}$$

(b)



Total 10 marks

[1]

[2]

- 4. The function f is defined by $f(x) = ln(1 + e^x)$
 - (a) Use Maclaurin's theorem to show that when f(x) is expanded in ascending powers of x:

(i) the first three terms are
$$ln2 + \frac{1}{2}x + \frac{1}{8}x^2$$
. [6]

(ii) the coefficient of
$$x^3$$
 is zero. [3]

(b) Hence write down the first two non-zero terms in the expansion, in ascending powers of x, of $\ln\left(\frac{1+e^x}{2}\right)$. [1] Total 10 marks 5. (a) The coefficient of x^2 in the expansion of $\left(x + \frac{1}{ax}\right)^8$ is 7. Find the possible value of a. [3]

(b) (i) Find the binomial expansion of $(1 + 6x)^{\frac{2}{3}}$ up to and including the term in x^2 . [2]

- (ii) Find the binomial expansion of $(8 + 6x)^{\frac{2}{3}}$ up to and including the term in x^2 . [3]
- (iii) Use your answer from (b) (ii) to find an estimate for $\sqrt[3]{100}$ in the form $\frac{a}{b}$ where *a* and *b* are integers. [2]

Total 10 marks

6. The equation

$$24x^3 + 36x^2 + 18x - 5 = 0$$

has one real root α .

- (a) Show that α lies in the interval 0.1 < x < 0.2. [3]
- (b) Starting from the interval 0.1 < x < 0.2, use interval bisection twice to obtain and interval of width 0.025 within which α must lie. [3]
- (c) Taking $x_1 = 0.2$ as a first approximation of α , use the Newton-Raphson method to find a second approximation x_2 of α . Give your answer to 4 decimal places. [4]

Total 10 marks