HARRISON COLLEGE INTERNAL EXAMINATION, MARCH 2015
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS
UNIT 1 - TEST 3

TIME: 1 Hour & 20 minutes

This examination paper consists of 3 printed pages.
The paper consists of 3 questions.
The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Number your questions carefully and do **NOT** write your solutions to different questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Electronic calculator (non-programmable, non-graphical)

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1. (a) Find $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$ [3]

(b) Find $\lim_{x\to 0}\frac{\sin(\frac{x}{5})}{2x}$ [3]

(c) The function *f* on ℝ is defined by

$$f(x)=\begin{cases}\frac{2x^2-x-15}{x-3}, & if\ x\neq 3\\ kx-1, & if\ x=3\end{cases}$$

Find the value of the constant *k* that makes *f* continuous at *x* = 3. [5]

(d) Let $y=x^{-3}$. Using first principles, find $\frac{dy}{dx}$. [5]

TOTAL 16 marks

2. (a) Find $f^{/}(x)$ when:

(i) $f(x) = \sqrt{(x^3 - 2x)}$ [3]

(ii) $f(x) = \frac{2x+1}{\sin 3x}$ [3]

(b) A manufacturer asks for a cylindrical tub to be constructed to contain a volume of 1 000 m^3. The tub is to be opened at the top and is to be made of material 1 cm in thickness. Let *R* be the **internal** radius and *h* be the **internal** height of the tub.

(i) Express *h* in terms of *R*. [2]

(ii) Show that the internal surface area *A* m^2 is given by

$A = \frac{2000}{R} + \pi R^2$ [2]

(iii) Hence determine the value of *R* which minimises the amount of material to be used. [3]

(c) A curve is defined by the parametric equations

$x = 3 - 4t \qquad y = 1 + \frac{2}{t}$

(i) Find $\frac{dy}{dx}$ in terms of *t*. [4]

(ii) Find the equation of the tangent to the given curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers. [4]

TOTAL 21 marks

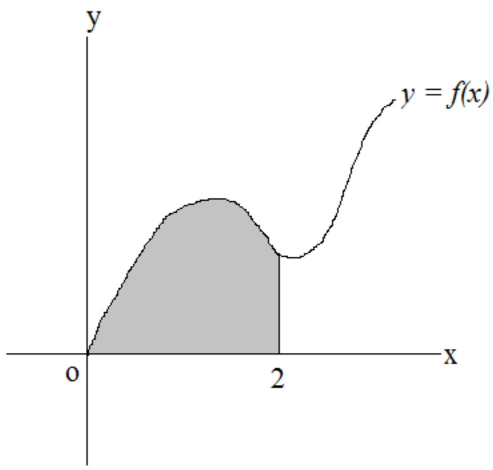
3. (a) The gradient of a curve is given by $\frac{dy}{dx} = (2x+1)^{-2}$. The point $(1, 1)$ lies on the curve. Find the equation of the curve. [4]

(b) (i) Find $\int_0^1 \cos(1 - 3x)\, dx$. Give your answer to 2 decimal places. [4]

(ii) Using the substitution $u = x^2 + 2$, find

$\int_1^2 x(x^2+2)^3\, dx$ [5]

3. (c) Find the shaded area in the diagram below which is bounded by the graph of $f(x) = \sin 2x + \sqrt{x}$, the x-axis and the lines $x = 0$ and $x = 2$.
Give your answer to 2 decimal places [5]



(d) Solve the differential equation $\frac{dy}{dx} = 5\frac{x^2}{y}$ given that $y = 3$ when $x = 3$. [5]

TOTAL 23 marks

End of test