CAPE Pure Mathematics SBA 2018
Unit I – Module 2

1) $4\tan^2\theta - 3\sec\theta = -3$

$4(\sec^2\theta - 1) - 3\sec\theta = -3$ (1)

$4\sec^2\theta - 3\sec\theta - 4 + 3 = 0$

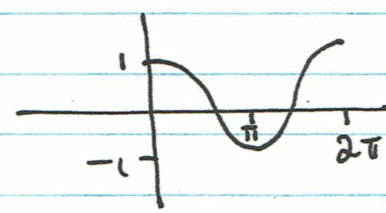
$4\sec^2\theta - 3\sec\theta - 1 = 0$ (1)

$(4\sec\theta + 1)(\sec\theta - 1) = 0$

$\boxed{7}$

$\sec\theta = -\frac{1}{4}$ (1) OR $\sec\theta = 1$ (1)

$\cos\theta = 1$ (1)



$\cos\theta = -4$

not valid

$\theta = 0, 2\pi$ (1) (1)

2) $RHS = \dfrac{\cot A - \tan A}{\cot A + \tan A}$

$$= \frac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\cos A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\cos A}} = \frac{\dfrac{\cos^2 A - \sin^2 A}{\sin A\cos A}}{\dfrac{\cos^2 A + \sin^2 A}{\sin A\cos A}}$$ (1) (1) (1) (1)

$\boxed{6}$

$$= \frac{\cos^2 A - \sin^2 A}{\cancel{\sin A\cos A}} \times \frac{\cancel{\sin A\cos A}}{\cos^2 A + \sin^2 A}$$ (1)

$$= \frac{\cos^2 A - \sin^2 A}{1} = \cos 2A$$ (1)

$= LHS$

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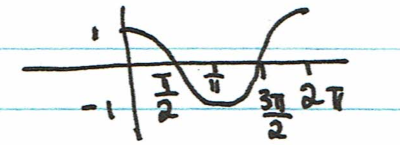
3) $\cos 3A + \cos 5A = 0$

using the factor formula



$$2\cos\left(\frac{3A+5A}{2}\right)\cos\left(\frac{3A-5A}{2}\right) = 0$$

$$2\cos 4A \cos(-A) = 0$$
1 1

$\cos 4A = 0$ OR $\cos(-A) = 0$

$\cos^{-1} 0 = \frac{\pi}{2}$ 1

Same as $\cos A = 0$

$\cos^{-1} 0 = \frac{\pi}{2}$ 1

6

general solution

$4A = 2n\pi \pm \frac{\pi}{2}$ OR general solution

$A = 2n\pi \pm \frac{\pi}{2}$ 1

$A = \frac{1}{2}n\pi \pm \frac{\pi}{8}$ 1

4) $4\cos x - \sqrt{3}\sin x \equiv R\cos(x+\alpha)$

$4\cos x - \sqrt{3}\sin x \equiv R[\cos x\cos\alpha - \sin x\sin\alpha]$

$4 = R\cos\alpha$

$\sqrt{3} = R\sin\alpha$

$\frac{\sqrt{3}}{4} = \frac{\sin\alpha}{\cos\alpha}$

$\tan\alpha = \frac{\sqrt{3}}{4}$

$\alpha = \tan^{-1}\frac{\sqrt{3}}{4}$

$= 23.4^0$ 1

$16 = R^2\cos^2\alpha$

$3 = R^2\sin^2\alpha$

$19 = R^2(\cos^2\alpha + \sin^2\alpha)$

$R = \sqrt{19}$ 1

$\therefore 4\cos x - \sqrt{3}\sin x = \sqrt{19}\cos(x + 23.4^0)$

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4) cont'd

Minimum occurs when $x + 23.4^\circ = 180^\circ$

$x = 180^\circ - 23.4^\circ$

$= 156.6^\circ$ (1)

$4\cos x - \sqrt{3}\sin x + 3 = 0$

$\sqrt{19}\cos(x + 23.4^\circ) + 3 = 0$

7

$\cos(x + 23.4^\circ) = \dfrac{-3}{\sqrt{19}}$ (1)

$\frac{S \mid A}{T \mid C}$

$\cos^{-1}\left(\dfrac{3}{\sqrt{19}}\right) = 46.5^\circ$ (1)

$x + 23.4^\circ = 180^\circ - 46.5^\circ, \ 180^\circ + 46.5^\circ$

$= 133.5^\circ, 226.5^\circ$

$x = 133.5^\circ - 23.4^\circ, \ 226.5^\circ - 23.4^\circ$

$= 110.1^\circ, \ 203.1^\circ$ (1) (1)

5) $x = 3t + 2$ $\qquad y = t^2 + 5$

$\dfrac{x-2}{3} = t$ (1)

* $y = \left(\dfrac{x-2}{3}\right)^2 + 5$ (1)

The form of the Cartesian equation was not specified so full marks were awarded at this point.

5

$y = \dfrac{(x-2)^2}{9} + 5$ Multiply throughout by 9

$9y = (x-2)^2 + 45$ (1)

$9y = x^2 - 4x + 4 + 45$ (1)

Cartesian equation

$9y = x^2 - 4x + 49$ (1)

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6)

$$x^2 + y^2 - 5x + 3y - 4 = 0$$
$$x^2 + y^2 - 4x + 6y - 12 = 0$$
subtract

$$-x - 3y + 8 = 0$$
$$x = 8 - 3y \quad (1)$$

sub for x into one of the equations

(1) for substitution

$$(8-3y)^2 + y^2 - 5(8-3y) + 3y - 4 = 0$$

$$64 - 48y + 9y^2 + y^2 - 40 + 15y + 3y - 4 = 0 \quad (1)$$

$$10y^2 - 30y + 20 = 0 \quad (1)$$

divide throughout by 10

$$y^2 - 3y + 2 = 0$$
$$(y-2)(y-1) = 0$$
$$y = 2 \quad (1) \quad \text{OR} \quad y = 1 \quad (1)$$

$\boxed{8}$

when $y = 2$, $x = 8 - 3(2)$
$= 2 \quad (1)$

when $y = 1$, $x = 8 - 3(1)$
$= 5 \quad (1)$

points of intersection are $(2,2)$ and $(5,1)$

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7) $x^2 + y^2 + 8x - 9 = 0$

$x^2 + 8x + y^2 - 9 = 0$

$(x+4)^2 - 16 + (y-0)^2 - 9 = 0$

$(x+4)^2 + (y-0)^2 = 25$ 1

centre of circle is $(-4, 0)$ radius $= 5$
1

gradient of normal at $(0,-3) = \dfrac{0-(-3)}{-4-0} = \dfrac{-3}{4}$
1

gradient of tangent $= \frac{4}{3}$ 1

5 equation of tangent looks like $y = \frac{4}{3}x + c$

sub $(0,-3)$ to find c

$c = -3$

equation of tangent is $y = \frac{4}{3}x - 3$ 1

OR $3y = 4x - 9$

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8) i) $\vec{OQ} = \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$ $\vec{OR} = \begin{pmatrix} 11 \\ 3 \\ 5 \end{pmatrix}$

$$\vec{QR} = \vec{QO} + \vec{OR}$$

$$= \begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix} + \begin{pmatrix} 11 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 14 \\ 0 \\ 3 \end{pmatrix} \quad 1$$

vector equation of plane $\underset{\sim}{r} \cdot \begin{pmatrix} 14 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 14 \\ 0 \\ 3 \end{pmatrix}$ 1

$\boxed{4}$

$$1 \quad \underset{\sim}{r} \cdot \begin{pmatrix} 14 \\ 0 \\ 3 \end{pmatrix} = 28 + 3$$

$$\underset{\sim}{r} \cdot \begin{pmatrix} 14 \\ 0 \\ 3 \end{pmatrix} = 31 \quad 1$$

ii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 14 \\ 0 \\ 3 \end{pmatrix} = 31$

$\boxed{2}$

Cartesian equation is $14x + 3z = 31$

1 1

iii) $\left| \begin{pmatrix} 14 \\ 0 \\ 3 \end{pmatrix} \right| = \sqrt{14^2 + 3^2}$

$$= \sqrt{205} \quad 1$$

$\boxed{2}$

$$\therefore \quad \underset{\sim}{r} \cdot \begin{pmatrix} \frac{14}{\sqrt{205}} \\ 0 \\ \frac{3}{\sqrt{205}} \end{pmatrix} = \frac{31}{\sqrt{205}} \quad 1$$

so distance from origin to plane is $\frac{31}{\sqrt{205}}$ units

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9) i) $\underset{\sim}{r} = \begin{pmatrix} 3 \\ -1 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$ $\therefore$ vector parallel to line is $\begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$

$\mu = \dfrac{x+5}{2}$ $\quad y - 3 = \mu$ $\quad \dfrac{z+1}{-1} = \mu$

$y = \mu + 3$

$2\mu - 5 = x$ $\quad z = -\mu - 1$

$\underset{\sim}{r} = \begin{pmatrix} -5 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ $\therefore$ vector parallel to line is $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ 1

$$\begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$$8 - 2 - 3 = \sqrt{16+4+9}\ \sqrt{4+1+1}\ \cos\theta$$

$$\frac{3 \ ①}{\sqrt{29}\sqrt{6} \ ①} = \cos\theta$$

$\boxed{4}$ $\theta = \cos^{-1}\left(\dfrac{3}{\sqrt{174}}\right) = 76.9°$ 1

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9 ii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3+4\lambda \\ -1-2\lambda \\ 6+3\lambda \end{pmatrix}$ $\qquad$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5+2\mu \\ 3+\mu \\ -1-\mu \end{pmatrix}$

$3+4\lambda = -5+2\mu$

$-1-2\lambda = 3+\mu \qquad \times 2$

$-2-4\lambda = 6+2\mu$

$1 = 1+4\mu$

$\mu = 0$ 1

4

$3+4\lambda = -5$

$4\lambda = -8$

$\lambda = -2$ 1

use $6+3\lambda = -1-\mu$ to check

$6+3(-2) = -1-0$ 1

The light beams do not intersect. 1

Reword 9 ii) Determine the coordinates of the point of intersection, if any, of these two light beams.