HARRISON COLLEGE INTERNAL EXAMINATION 2021 PREVIEW

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 1 – TEST 2

Time: 1 hour and 20 minutes

1. a) Prove that

$$\sin\left(\frac{\pi}{2} + \theta\right)\cos\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} + \theta\right)\sin\left(\frac{\pi}{2} - \theta\right) = 0$$

[3 marks]

b) Show that

$$(\sin x + cosec\ x)^2 \equiv sin^2 x + cot^2 x + 3$$

[4 marks]

c) Find the general solution of the equation

$$2\ cos^2 x = \sin x + 1$$

[7 marks]

$$\left[n\pi + (-1)^n \frac{\pi}{6}\right] \quad and \quad \left[n\pi - (-1)^n \frac{\pi}{2}\right]$$

d) i) Express $3\cos x - 2\sin x$ in the form $r\cos(x + \alpha)$ where $r > 0\ and\ 0^0 \leq x \leq 90^0$.

[4 marks]

$$\left[\sqrt{13}\cos(x + 33.7^0)\right]$$

ii) Hence, solve the equation $3\cos x - 2\sin x = 3, for\ 0^0 \leq x \leq 360^0$.

[6 marks]

$$[0^0, 292.6^0]$$

Total 24 marks

2. A circle, C, has equation $x^2 + y^2 + 6x + 2y + 8 = 0$.

   i)   Find the radius and the coordinates of the centre of the circle.

   [4 marks]

   $$\left[radius = \sqrt{18}, \; centre \; (-3, -1)\right]$$

   ii)   Find the equation of the tangent to the circle at the point $P(-2, -2)$.

   [4 marks]

   $$[\; y = x \;]$$

   iii)   The point P is at one end of the diameter of the circle C. Find the coordinates of the point Q, at

   the opposite end of the diameter from P.

   [3 marks]

   $$[\; Q(-4,0) \;]$$

   Total 11 marks

3. a) A curve, C, has parametric equations

$$x = 3 \sin t \; and \; y = 3 \cos t, 0 \le \theta < \frac{\pi}{2},$$

Find the Cartesian equation of C.

[3 marks]

$$[x^2 + y^2 = 9]$$

b) Find the points of intersection of the curve

$$C_1 \text{ with parametric equations } x = 2t^2 - 1 \text{ and } y = 5t$$

and the line

$$L_1 \text{ with Cartesian equation } 2y = 5x + 5$$

[7 marks]

$$[(-1,0) \text{ and } (1,5)]$$

Total 10 marks

4. a) Find the angle between the lines with equations

$$r = \begin{pmatrix} 3 \\ -5 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -1 \\ -3 \end{pmatrix} \text{ and } \frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+8}{-6}$$

[5 marks]

$$[\ 81.2^0\ ]$$

b) Find the point of intersection of the lines with equations

$$r_1 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \text{ and } r_2 = \begin{pmatrix} -1 \\ 2 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

[6 marks]

$$\left[\left(\frac{5}{4},\ -\frac{1}{4},\ -\frac{1}{2}\right)\right]$$

c) Find the Cartesian equation of the plane passing through the point $A(4, 3, 3)$ and parallel to the

plane with equation $2x - y + 2z = 10$.

[4 marks]

$$[2x - y + 2z]$$

Total 15 marks