HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2019
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT I – TEST 1
1 hour 20 minutes

This examination paper consists of **2** printed pages.
This paper consists of **9** questions.
The maximum mark for this examination is **60**.

<u>INSTRUCTIONS TO CANDIDATES</u>
(i) Write your name clearly on each sheet of paper used
(ii) Answer **ALL** questions
(iii) Number your questions identically as they appear on the question paper and do **NOT write your solutions to different questions** beside each other
(iv) Unless otherwise stated in the question, any numerical answer that is not <u>exact</u>, **MUST** be written correct to <u>three</u> (3) significant figures

<u>EXAMINATION MATERIALS ALLOWED</u>
(a) Mathematical formulae
(b) Scientific calculator (non-programmable, non-graphical)

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1) Given that $\boldsymbol{p}$ and $\boldsymbol{q}$ are propositions, use the <u>algebra of propositions</u> to prove
$\sim(\boldsymbol{p} \vee \sim\boldsymbol{q}) \vee (\sim\boldsymbol{p} \wedge \sim\boldsymbol{q}) \equiv \sim\boldsymbol{p}$ [4]

Total: 4 marks

2) Prove that for all $x \in \boldsymbol{R}, y \in \boldsymbol{R}; x^2 + y^2 \geq 2xy$ [3]

Total: 3 marks

3) Without the use of a calculator, find the <u>EXACT</u> value of $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} + \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ [4]

Total: 4 marks

4) Prove by mathematical induction that $\sum_{r=1}^{n} 2(3^{r-1}) = 3^n - 1 \ \forall\, n \in \boldsymbol{N}$. [7]

Total: 7 marks

5) Given that $(x - 1)$ is a factor of the function $f(x) = x^3 + px^2 + 5x - 12$, where $p \in \boldsymbol{R}$, find

(i) the value of p [2]

(ii) the remaining factors of $f(x)$. [5]

Total: 7marks

Please Turn Over

6) (a) Solve for x, $3log_5 x - 5 = 2log_x 5$. [6]

(b) Solve for x the following equation $e^{x} + 2e^{-x} = 3$. [4]

Total: 10 marks

7) The population, $P(t)$, of fish found in a swamp after t days is modelled by $P(t) = 1000e^{0.04t}$

(i) Determine for the swamp

(a) the initial population [2]
(b) the population after 8 days [2]

(ii) The length of time, in days, for which the population is expected to reach 2500. [4]
Total: 8 marks

8) Find the range of values of x for which $\left|\frac{2x+9}{x-3}\right| \leq 7$, $x \neq 3$. [5]
Total: 5 marks

9) If α, β and γ are the roots of the equation $2x^3 - 11x^2 + 4x + 5 = 0$

(i) find the values of $\alpha + \beta + \gamma$, $\alpha\beta + \alpha\gamma + \beta\gamma$ and $\alpha\beta\gamma$ [3]
(ii) hence, or otherwise, find the equation with roots $\alpha + 1, \beta + 1$ and $\gamma + 1$. [9]
Total: 12 marks

END OF TEST

HARRISON COLLEGE INTERNAL EXAMINATIONS 2019: CAPE PURE MATHEMATICS

[UNIT I – TEST 1]

SOLUTIONS AND MARK SCHEME

Question	Working	Marks & Comments
1)	$\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$ Proof: LHS $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q)$ $= (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ $= \sim p \wedge (q \vee \sim q)$ $= \sim p \wedge (1)$ $= \sim p$	 1 1 1 1 Total = 4 marks
2)	Prove that for all $x \in \boldsymbol{R}, y \in \boldsymbol{R}; x^2 + y^2 \geq 2xy$ Proof: $(x-y)^2 \geq 0$ for all $x \in \boldsymbol{R}, y \in \boldsymbol{R}$ $x^2 + y^2 - 2xy \geq 0$ $x^2 + y^2 \geq 2xy$	 1 1 1 Total = 3 marks
3)	$\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} + \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ $= \frac{(\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2}) + (\sqrt{5}-\sqrt{2})(\sqrt{5}-\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$ $= \frac{(5+2\sqrt{5}\sqrt{2}+2)+(5-2\sqrt{5}\sqrt{2}+2)}{5-2}$ $= \frac{14}{3}$	 [1 + 1] Numerator + Denominator 1 1 **CAO** Total = 4 marks

4)	Prove by mathematical induction that $\sum_{r=1}^{n} 2(3^{r-1}) = 3^n - 1 \ \forall\, n \in N$. Let P_n be the proposition "$\sum_{r=1}^{n} 2(3^{r-1}) = 3^n - 1 \ \forall\, n \in N$" <u>Basic Step</u> $r = 1$: LHS $= 2(3^{1-1}) = 2$ $n = 1$: RHS $= 3^1 - 1) = 2$ $\therefore P_1$ is true <u>Inductive Step</u> Assume P_n is true for $n = k$ i.e. $P_k \equiv$ "$\sum_{r=1}^{k} 2(3^{r-1}) = 3^k - 1 \ \forall\, k \in N, k > 1$" We are required to show that $P_{k+1} \equiv \sum_{r=1}^{k+1} 2(3^{r-1}) = 3^{k+1} - 1$ Now $P_{k+1} = P_k + (k+1)st$ term $= (3^k - 1) + 2(3^{k+1-1})$ $= 3^k - 1 + 2(3^k)$ $= 3(3^k) - 1$ $= 3^{k+1} - 1$ As required P_k implies P_{k+1} i.e. P_1 implies P_2, P_2 implies P_3, …, P_{n-1} implies P_n Hence, by Mathematical Induction, P_n is true.	1 1 1 1 1 1 1 Total = 7 marks
5)(i)	$f(x) = x^3 + px^2 + 5x - 12$ $f(1) = 0$ implies $(1)^3 + p(1)^2 + 5(1) - 12 = 0$ $p = 6$	1 1 Total = 2 marks
5)(ii)	$f(x) = x^3 + 6x^2 + 5x - 12$ $f(x) = (x - 1)(x^2 + 7x - 12)$ $f(x) = (x - 1)(x + 3)(x + 4)$	1, for quotient 1 + 1, for process of getting the quotient, and factorizing it. 2 marks for remaining factors Total = 5 marks

6)(a)	$3log_5x - 5 = 2log_x5$ $3log_5x - 5 = \frac{2}{log_x5}$ $3(log_x5)^2 - 5log_x5 - 2 = 0$ $(3log_x5 + 1)(log_x5 - 2) = 0$ $log_x5 = -\frac{1}{3}, log_x5 = 2$ $x = \frac{1}{\sqrt[3]{5}}, x = 25$	1 1 1+1 1+1 Total = 6 marks
6(b)	$e^x + 2e^{-x} = 3$ $e^x + \frac{2}{e^x} - 3 = 0$ $e^{2x} - 3e^x + 2 = 0$ $(e^x - 2)(e^x - 1) = 0$ $e^x = 2, e^x = 1$ $x = \ln 2, x = 0$	1 1 1+1 Total = 4 marks

7) (i)	(a) the initial population at $t = 0$, $P(0) = 1000e^0$ = 1000 (b) for $t = 8$, $P(8) = 1000 \times e^{0.04(8)}$ = 1377	1 1 1 1	Total = 4 marks
7) (ii)	$2500 = 1000 \times e^{0.04t}$ $2.5 = e^{0.04t}$ $\ln 2.5 = 0.04t$ 23.9 days = t	1 1 1 1	Total = 4 marks

8)	$\left\|\frac{2x+9}{x-3}\right\| \le 7, x \ne 3.$ $\|2x+9\| \le 7\|x-3\|$ $(2x+9)^2 \le 49(x-3)^2$ $4x^2 + 36x + 81 \le 48x^2 - 294x + 441$ $0 \le 45x^2 - 330x + 360$ $0 \le 3x^2 - 22x + 24$ $0 \le (3x-4)(x-6)$ $x \le \frac{4}{3}, x \ge 6$ **FT**	1 1 1 1 + 1 Total = 5 marks
9)(i)	$2x^3 - 11x^2 + 4x + 5 = 0$ $\alpha + \beta + \gamma = -\frac{b}{a} = \frac{11}{2}$ $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = 2$ $\alpha\beta\gamma = -\frac{d}{a} = -\frac{5}{2}$	1 1 1 Total = 3 marks
9)(ii)	Sum; $(\alpha+1) + (\beta+1) + (\gamma+1)$ $= \alpha + \beta + \gamma + 3$ $= \frac{17}{2}$ Sum Pairwise; $(\alpha+1)(\beta+1) + (\alpha+1)(\gamma+1) + (\beta+1)(\gamma+1)$ $= (\alpha\beta + \alpha\gamma + \beta\gamma) + 2(\alpha + \beta + \gamma) + 3$ $= 2 + 2(\frac{11}{2}) + 3$ $= 16$ Product; $(\alpha+1)(\beta+1)(\gamma+1)$ $= \alpha\beta\gamma + (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) + 1$ $= -\frac{5}{2} + 2 + \frac{11}{2} + 1$ $= 6$ Required equation; $x^3 - \frac{17}{2}x^2 + 16x - 6 = 0$ OR $2x^3 - 17x^2 + 32x - 12 = 0$	1 1 1 1 1 1 1 1 1 Total = 9 marks

9)(ii) Alternative Solution	$2x^3 - 11x^2 + 4x + 5 = 0$ $x = (\alpha + 1),\ (\beta + 1),\ (\gamma + 1)$ i.e. $x = X + 1 \rightarrow x - 1 = X$ $2(x-1)^3 - 11(x-1)^2 + 4(x-1) + 5 = 0$ $2(x^3 - 3x^2 + 3x - 1) - 11(x^2 - 2x + 1) + 4x + 1 = 0$ $2x^3 - 6x^2 + 6x - 2 - 11x^2 + 22x - 11 + 4x + 1 = 0$ $2x^3 - 17x^2 + 32x - 12 = 0$	 1 1 1 1 1 1-mark x 4 Total = 9 marks