HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2019
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT I –TEST 2
1 hour 20 minutes

This examination paper consists of **3** printed pages.

This paper consists of **6** questions.

The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES

(i) Write your name clearly on each sheet of paper used

(ii) Answer **ALL** questions

(iii) Number your questions identically as they appear on the question paper and do **NOT write your solutions to different questions** beside each other

(iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED
(a) Mathematical formulae
(b) Scientific calculator (non-programmable, non-graphical)

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1) Prove that $\frac{\cos A}{1-\sin A}+\frac{1-sinA}{cosA}\equiv\frac{2}{cosA}$. [5]

Total 5 marks

2) (i) Prove that $\frac{\sin 9A-\sin 5A}{\cos 9A+\cos 5A}\equiv\tan 2A$. [4]

(ii) Hence, find the general solution of the equation $\frac{\sin 9A-\sin 5A}{\cos 9A+\cos 5A}=\sqrt{3}$. [4]

Total 8 marks

Please Turn Over

3) (i) Express $cos\ x - 3\sin x$ in the form $\boldsymbol{R}cos(x + \alpha)$, where $\boldsymbol{R} > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$,

giving the exact value of $\boldsymbol{R}$, and the value of α correct to 3 decimal places. [6]

(ii) Hence, solve the equation $cos\ x - 3\sin x = \sqrt{5}$ for $0 \leq x \leq 2\pi$, giving your

answers correct to 1 decimal place. [6]

Total 12 marks

4) Find the coordinates of the points of intersection of the two circles

$x^2 + y^2 + 8x + 2y - 2 = 0$ and $x^2 + y^2 + 8x + 4y + 2 = 0$, giving your answers

in exact form [7]

Total 7 marks

5) (a) (i) Find the Cartesian equation of the curve, *C*, represented by the parametric equations

$x = 7 + 5\cos\theta$ and $y = -2 + 5\sin\theta$. [4]

(ii) Describe fully, the locus of *C*. [3]

(b) A line, $\boldsymbol{l}$, passes through the point $\boldsymbol{P}$ (2, − 3, 4) and is parallel to the vector $5\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$.

Find, for the line $\boldsymbol{l}$

(i) its vector equation [2]

(ii) its cartesian equations [3]

(c) Find the angle between the lines with equations

$\boldsymbol{r}_1 = (5 + 6\lambda)\,\boldsymbol{i} + (2 - 5\lambda)\,\boldsymbol{j} + (-8 + 2\lambda)\,\boldsymbol{k}$ and

$\boldsymbol{r}_2 = (-4 - 8\mu)\,\boldsymbol{i} + (-7 + \mu)\,\boldsymbol{j} + (6 - 4\mu)\,\boldsymbol{k}$, giving your answer correct to 1 decimal

place. [5]

Total 17 marks

Please Turn Over

6) A plane contains three non-collinear points $\boldsymbol{A}\ (-1, 2, -5)$, $\boldsymbol{B}\ (7, -4, 6)$ and $\boldsymbol{C}\ (3, 2, -4)$

(i) Determine the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ [2]

(ii) Prove that the vector $-\boldsymbol{i} + 6\boldsymbol{j} + 4\boldsymbol{k}$ is normal to the plane [3]

(iii) Hence, obtain the Cartesian equation of the plane [3]

(iv) Find the perpendicular distance of the plane from the origin. [3]

Total 11 marks

END OF TEST

HARRISON COLLEGE INTERNAL EXAMINATIONS 2019: CAPE PURE MATHEMATICS

[UNIT I – TEST 2]

SOLUTIONS AND MARK SCHEME

Question	Working	Marks and Comments
1)	Prove that $\frac{\cos A}{1-\sin A}+\frac{1-sinA}{cosA}\equiv\frac{2}{cosA}$. Proof: LHS $\frac{\cos A}{1\ -\ \sin A}+\frac{1-sinA}{cosA}$ $=\frac{cos\cos A+(1-sinA)(1-sinA)}{(1\ -\ \sin A)cosA}$ $=\frac{cos^2A+1-2SinA+Sin^2A}{(1\ -\ \sin A)cosA}$ $=\frac{2-2SinA}{(1\ -\ \sin A)cosA}$ $=\frac{2(1-SinA)}{(1\ -\ \sin A)cosA}$ $=$ $\frac{2}{cosA}$	1, Numerator 1, Denominator 1 1 1 Total = 5 marks
2) (i)	Prove that $\frac{\sin 9A-\sin 5A}{\cos 9A+\cos 5A}\equiv\tan 2A$. Proof: LHS $\frac{\sin 9A\ -\sin 5\,A}{\cos 9A\ +\ \cos 5\,A}$ $=\frac{2\cos\frac{14A}{2}sin\frac{4A}{2}}{2\cos\frac{14A}{2}cos\frac{4A}{2}}$ $=\frac{2\cos\ 7Asin2A}{2\cos\ 7Acos2A}$	1, Numerator 1, Denominator 1 1 Total = 4 marks

	$= \tan 2A$	
2 (ii)	$\dfrac{\sin 9A - \sin 5A}{\cos 9A + \cos 5A} = \sqrt{3}$ $\tan 2A = \sqrt{3}$ Principal value, $\frac{\pi}{3}$ General solution $2A = n\pi + \frac{\pi}{3}$ $A = \frac{n\pi}{2} + \frac{\pi}{6}$	 1 1 1 1 Total = 4 marks
3) (i)	$\cos x - 3\sin x \equiv R\cos(x + \alpha)$ $R\cos\alpha = 1$ $R\sin\alpha = 3$ $R = \sqrt{10}$ $\tan \alpha = 3$ $\alpha = 1.249$ rad $\cos x - 3\sin x \equiv \sqrt{10}\cos(x + 1.249)$	 1 1 1 1 1 1 Total = 6 marks
3) (ii)	$\cos x - 3\sin x = \sqrt{5}$ $\sqrt{10}\cos(x + 1.249) = \sqrt{5}$ $\cos(x + 1.249) = \dfrac{1}{\sqrt{2}}$ Principal value, 0.785 rad General solution $(x + 1.249) = 2n\pi \pm 0.785$ $x = 2n\pi \pm 0.785 - 1.249$ $x = 4.2$ rad, 5.8 rad	 1 1 1 1 2 Total = 6 marks
4)	$x^2 + y^2 + 8x + 2y - 2 = 0$ Eqn 1 $\underline{x^2 + y^2 + 8x + 4y + 2 = 0}$ Eqn 2 $2y + 4 = 0$ $y = -2$ sub. Into Eqn 1	 1

	$x^2 + 8x - 2 = 0$ $x = \dfrac{-8 \pm \sqrt{72}}{2}$ $x = -4 \pm 3\sqrt{2}$ Points of intersection, $(-4 - 3\sqrt{2}, -2)$ and $(-4 + 3\sqrt{2}, -2)$	1 1 2 1 1 Total = 7 marks
5) (a)(i)	$x = 7 + 5\cos\theta \rightarrow \frac{x-7}{5} = cos\theta$ $y = -2 + 5\sin\theta \rightarrow \frac{y+2}{5} = sin\theta$ $\cos^2\theta + \sin^2\theta = (\frac{x-7}{5})^2 + (\frac{y+2}{5})^2$ $5^2 = (x-7)^2 + (y+2)^2$	1 1 1 1 Total = 4 marks
5) (a)(ii)	C represents a Circle, centre (7, – 2), radius 5	1+1+1 Total = 3 marks
5 (b)(i)	vector equation, $\boldsymbol{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}$	1, for point 1, for direction vector Total = 2 marks
5 (b)(ii)	cartesian equations, $\frac{x-2}{5} = \frac{y+3}{2} = \frac{z-4}{-1}$	1+1+1 Total = 3 marks
5 (c)	$\boldsymbol{r}_1 = (5 + 6\lambda)\,\boldsymbol{i} + (2 - 5\lambda)\,\boldsymbol{j} + (-8 + 2\lambda)\,\boldsymbol{k}$ $\boldsymbol{r}_2 = (-4 - 8\mu)\,\boldsymbol{i} + (-7 + \mu)\,\boldsymbol{j} + (6 - 4\mu)\,\boldsymbol{k}$ $cos\theta = \dfrac{\begin{pmatrix} 6 \\ -5 \\ 2 \end{pmatrix} . \begin{pmatrix} -8 \\ 1 \\ -4 \end{pmatrix}}{\left\|\begin{pmatrix} 6 \\ -5 \\ 2 \end{pmatrix}\right\| \left\|\begin{pmatrix} -8 \\ 1 \\ -4 \end{pmatrix}\right\|}$ $= \frac{-48-5-8}{(\sqrt{65})(\sqrt{81})}$ $\theta = cos^{-1}\left(-\dfrac{61}{9\sqrt{65}}\right)$ $\theta = 32.8^0$ or 147.2^0 OR 0.6 rad or 2.6 rad	1, numerator 1, denominator 1, numerator 1, denominator 1 Total = 5 marks

6) (i)	$\overrightarrow{AB} = \begin{pmatrix} 8 \\ -6 \\ 11 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} -4 \\ 6 \\ -10 \end{pmatrix}$	1 1 Total = 2 marks
6) (ii)	$\boldsymbol{n}.\overrightarrow{AB} = \begin{pmatrix} -1 \\ 6 \\ 4 \end{pmatrix}.\begin{pmatrix} 8 \\ -6 \\ 11 \end{pmatrix} = -8 - 36 + 44 = 0$ $\boldsymbol{n}.\overrightarrow{BC} = \begin{pmatrix} -1 \\ 6 \\ 4 \end{pmatrix}.\begin{pmatrix} -4 \\ 6 \\ -10 \end{pmatrix} = 4 + 36 - 40 = 0$ Since ***n*** is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{BC}$, it is perpendicular to the plane through ***A***, ***B*** and ***C***.	1 1 1 Total = 3 marks
6) (iii)	Cartesian equation, $\boldsymbol{r}.\begin{pmatrix} -1 \\ 6 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -5 \end{pmatrix}.\begin{pmatrix} -1 \\ 6 \\ 4 \end{pmatrix}$ $= 1 + 12 - 20$ i.e. $\boldsymbol{r}.\begin{pmatrix} -1 \\ 6 \\ 4 \end{pmatrix} = 7$ $-x + 6y + 4z = -7$ OR $x - 6y - 4z = 7$	1 + 1 [LHS + RHS] 1 Total = 3 marks
6) (iv)	$\boldsymbol{r.n} = \boldsymbol{a.n} = d$ $\boldsymbol{r}.\hat{\boldsymbol{n}} = \boldsymbol{a}.\hat{\boldsymbol{n}} = \frac{d}{\lvert\boldsymbol{n}\rvert}$ $\hat{\boldsymbol{n}} = \frac{1}{\sqrt{(-1)^2+6^2+4^2}}\begin{pmatrix} -1 \\ 6 \\ 4 \end{pmatrix} = \frac{1}{\sqrt{53}}\begin{pmatrix} -1 \\ 6 \\ 4 \end{pmatrix}$ $\boldsymbol{r}.\hat{\boldsymbol{n}} = \frac{-7}{\sqrt{53}}$ Perpendicular distance $= \left\lvert\frac{-7}{\sqrt{53}}\right\rvert$ $= \frac{7}{\sqrt{53}}$	 1 1 1 Total = 3 marks