HARRISON COLLEGE SBA EXAMINATIONS 2015 : PURE MATHEMATICS [UNIT 1 – TEST: 3]

SOLUTIONS AND MARK SCHEME

Question	Working	Marks & comments
1.(a)	$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$ $= \lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}$ $= \lim_{x\to 4}\frac{x-4}{(x-4)(\sqrt{x}+2)}$ $= \lim_{x\to 4}\frac{1}{\sqrt{x}+2}$ $= \frac{1}{4}$	1 rationalising the numerator OR factorizing denominator as $(\sqrt{x}+2)(\sqrt{x}-2)$ 1 simplifying expression 1 c.a.o Total = 3
(b)	$\lim_{x\to 0}\frac{\sin(\frac{x}{5})}{2x}$ let $u=\frac{x}{5}$, $\therefore x = 5u$ also as $x\to 0$ $u\to 0$ on substituting into given limit $\lim_{u\to 0}\frac{\sin(u)}{10u} = \frac{1}{10}\, lim_{u\to 0}\frac{sin(u)}{u}$ (or $\frac{1}{10}\lim_{x\to 0}\frac{\sin(\frac{x}{5})}{\frac{x}{5}}$) $= \frac{1}{10}\cdot 1 = \frac{1}{10}$	1 choosing substitution OR multiplying by limit expression by $\frac{\frac{x}{5}}{\frac{x}{5}}$. 1 simplifying and using $\lim_{x\to a}Kf(x) = K\lim_{x\to a}f(x)$ 1 using $\lim_{u\to 0}\frac{\sin(u)}{u} = 1$ Total = 3
(c)	For f to be continuous at $x = 3$ $\lim_{x\to 3}\frac{2x^2-x-15}{x-3} = f(3)$ $\lim_{x\to 3}\frac{(2x+5)(x-3)}{x-3} = 3k-1$ $\lim_{x\to 3} 2x+5 = 3k-1$ $11 = 3k-1$ $k = 4$	1 equating 1 factorising the numerator of $\lim_{x\to 3}\frac{2x^2-x-15}{x-3}$ 1 simplifying limit expression 1 substituting $x = 3$ 1 c.a.o Total = 5

1 (d)	$y = x^{-3}$	
	$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{(x+\delta x)^{-3} - x^{-3}}{\delta x}$	1 correctly expressing the derivative as a limit
	$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\frac{1}{(x+\delta x)^3} - \frac{1}{x^3}}{\delta x}$	
	$\frac{dy}{dx} = lim_{\delta x \to 0} \frac{\frac{x^3-(x+\delta x)^3}{(x+\delta x)^3 x^3}}{\delta x}$	1 expressing the numerator a single fraction
	$\frac{dy}{dx} = lim_{\delta x \to 0} \frac{\frac{x^3-x^3-3x^2\delta x-3x(\delta x)^2-(\delta x)^3}{(x+\delta x)^3 x^3}}{\delta x}$	1 expanding the terms in the numerator
	$\frac{dy}{dx} = lim_{\delta x \to 0} \frac{-3x^2-3x(\delta x) \quad -(\delta x)^2}{(x+\delta x)^3 x^3}$	1 simplifying the expression
	$= \frac{-3x^2}{x^6} \quad = -3x^{-4}$	1 substituting $\delta x = 0$ Total = 5

2(a)(i)	$f(x) = \sqrt{x^3 - 2x} = (x^3 - 2x)^{\frac{1}{2}}$ $f^{/}(x) = \frac{1}{2}(x^3 - 2x)^{-\frac{1}{2}}$ …. $f^{/}(x) = \frac{1}{2}(x^3 - 2x)^{-\frac{1}{2}} \times \ldots$ $f^{/}(x) = \frac{1}{2}(x^3 - 2x)^{-\frac{1}{2}} \times (3x^2 - 2)$	 1 1 1 Total =3
(a) (ii)	$f(x) = \dfrac{2x+1}{\sin 3x}$ $f^{/}(x) = \frac{\cdots}{(sin3x)^2}$ $f^{/}(x) = \frac{(sin3x)\times 2 - \cdots}{(sin3x)^2}$ $f^{/}(x) = \frac{(sin3x)\times 2-(2x+1)(cos3x)\times 3}{(sin3x)^2}$ $f^{/}(x) = \frac{2(sin3x)-(6x+3)(cos3x)}{(sin3x)^2}$	 1 using the quotient rule: squaring the denominator 1 $\sin(3x) \times 2$ 1 subtracting $(2x+1) \times$ the derivative of $sin3x$ Total =3
(b) (i)	Volume of cylinder = $\pi R^2 h = 1000$ $h = \frac{1000}{\pi R^2}$	1 equating formula for volume of cylinder to 1000 1 rearranging Total =2
(ii)	Surface area, $A = 2\pi Rh + \pi R^2$ $A = 2\pi R\frac{1000}{\pi R^2} + \pi R^2$ $A = \dfrac{2000}{R} + \pi R^2$	1 correct formula 1 substituting for *h* from (b) (i) Total =3

(iii)	$\frac{dA}{dR} = -\frac{2000}{R^2} + 2\pi R$ $-\frac{2000}{R^2} + 2\pi R = 0$ $R^3 = \frac{1000}{\pi}$ $R = \frac{10}{\sqrt{\pi}} = 6.83\ m$	1 differentiating A w.r.t. R 1 equating $\frac{dA}{dR}$ to 0 1 c.a.o Total =3
(c) (i)	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ $\frac{dy}{dt} = -\frac{2}{t^2}$ $\frac{dx}{dt} = -4 \ \rightarrow \frac{dt}{dx} = -\frac{1}{4}$ $\frac{dy}{dx} = \ -\frac{2}{t^2} \times -\frac{1}{4} = \frac{1}{2t^2}$	1 using the chain rule s.o.i. 1 derivative of *y* w.r.t *t* 1 derivative of *x* w.r.t *t* 1 c.a.o Total = 4
(c) (ii)	Gradient of tangent at *t=2*, $m = \frac{1}{2\times4} = \frac{1}{8}$ At *t* = 2, $x = 3 - 4(2) = -5$ $y = 1 + \frac{2}{2} = 2$ $(y - 2) = \frac{1}{8}(x + 5)$ $x - 8y + 21 = 0$	1 using $\frac{dy}{dx}$ from (c) (i) correctly to find the gradient of the tangent. 1 getting coordinates of point on curve 1 using equation of a straight line 1 expressing equation of line in required form Total = 4

3(a)	$\frac{dy}{dx} = (2x+1)^{-2}$ $y = \int (2x+1)^{-2}\,dx$ $= -\frac{1}{2(2x+1)} + constant$ Substitute $x = 1$ and $y = 1$ to find value of constant of integration. $c = \frac{7}{6}$ $y = \frac{7}{6} - \frac{1}{2(2x+1)}$	1 attempt to integrate $(2x+1)^{-2}$ 1 integrating $(2x+1)^{-2}$ correctly 1 s.o.i 1 c.a.o. Total = 4
(b)(i)	$y = \int_0^1 \cos(1-3x)\,dx$ $y = -\frac{\sin(1-3x)}{3}\Big\vert_0^1$ $y = -\frac{\sin(-2)}{3} - \left(-\frac{sin\,1}{3}\right)$ $y = 0.58$ (0.02 if working in degree measure)	1 integrating correctly 1 substituting limits 1 working in radian measure AND substituting limits correctly 1 Total = 4
(b) (ii)	$u = x^2 + 2$ $\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$ When $x = 1$ $u = 3$ $x = 2$ $u = 6$ So integral transforms to: $\int_3^6 \frac{u^3}{2}\,du$ $= \frac{u^4}{8}\Big\vert_3^6$ $= \frac{6^4}{8} - \frac{3^4}{8} = 151.875$ = 152 (to 3 s.f.)	1 differentiating 1 changing limits 1 substituting for *u* 1 integrating correctly 1 c.a.o Total =5

(c)	$Area = \int_0^2 sin2x + \sqrt{x}\, dx$ $= -\frac{\cos 2x}{2} + \cdots$ $= -\frac{cos2x}{2} + \frac{2}{3}x^{\frac{3}{2}} \Big\|_0^2$ $= (0.327 + 1.886) - (-0.5 - 0)$ = 2.71 (1.89 if working in degree measure)	1 writing area as an integral 1 correctly integrating $\sin 2x$ 1 correctly integrating $\sqrt{x}$ 1 working in radian measure AND substituting limits correctly. 1 Total = 5
(d)	$\frac{dy}{dx} = \frac{5x^2}{y}$ $y\, dy = 5x^2\, dx$ $\int y\, dy = \int 5x^2\, dx$ $\frac{y^2}{2} = \cdots$ $\frac{y^2}{2} = 5\frac{x^3}{3} + c$ Substitute $x = y = 3$ to determine value of *c* . $c = -\frac{81}{2}$ $y^2 = 10\frac{x^3}{3} - 81$ OR $3y^2 = 10x^3 - 243$	1 setting up integrals 1 integrating *y* correctly 1 integrating $5x^2$ correctly 1 substituting *x* = *y* =3 to determine *c* 1 correct value of c Total =5