HARRISON COLLEGE

**END OF YEAR EXAMINATION 2018**

**FOURTH YEAR MATHEMATICS**

**PAPER 02**

**DURATION: 2 Hours**

## INSTRUCTIONS TO CANDIDATES

1) This question paper consists of **FIVE** printed pages and 10 questions.
2) Write your name clearly on **EACH** sheet of paper used.
3) All questions are to be attempted.
4) All working must be clearly shown.
5) Number your responses carefully and identically (including any associated parts) as they appear on the question paper.

   DO NOT WRITE ANY responses beside each other.

6) Calculators are allowed.
7) If a numerical answer cannot be given **exactly**, and the accuracy required is not specified in the question, then in the case of an angle, it must be given correct to one (1) decimal, and in others cases it must be given correct to three (3) significant figures.
8) The maximum mark for this examination is 85.

**DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO**

**CSEC Mathematics**
**LIST OF FORMULAE**

Volume of Prism $V = Ah$ where $A$ is the area of a cross-section and $h$ is the perpendicular length.

Volume of Cylinder $V = \pi r^2 h$ where $r$ is the radius of the base and $h$ is the perpendicular height.

Volume of a right pyramid $V = \frac{1}{3}Ah$ where $A$ is the area of the base and $h$ is the perpendicular height.

Circumference $C = 2\pi r$ where $r$ is the radius of the circle.

Arc length $S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.

Area of a circle $A = \pi r^2$ where $r$ is the radius of the circle.

Area of a sector $A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.

Area of Trapezium $A = \frac{1}{2}(a+b)h$ where $a$ and $b$ are the lengths of the parallel sides and $h$ is the perpendicular distance between the parallel sides.

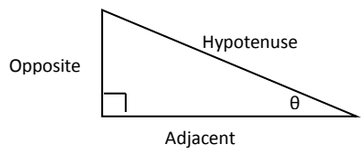
Roots of quadratic equations If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

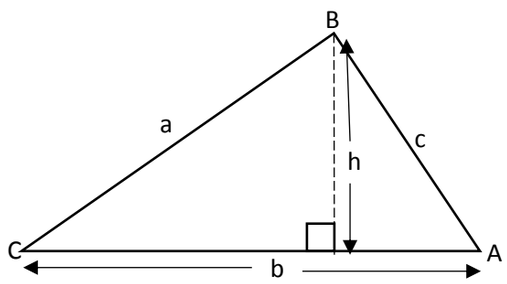
$\sin\theta = \frac{opposite \quad side}{hypotenuse}$

$\cos\theta = \frac{adjacent \quad side}{hypotenuse}$

$\tan\theta = \frac{opposite \quad side}{adjacent \quad side}$




Area of a triangle Area of $\Delta = \frac{1}{2}bh$ where $b$ is the length of the base and $h$ is the perpendicular height




Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

1. A Universal set, $U$, is defined as $U = \{25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37\}$.

   Sets $M \text{ and } N$ are subsets of $U$ such that $M = \{Odd\ numbers\} \text{ and } N = \{Multiples\ of\ 3\}$

   i) Draw a Venn diagram to represent the sets $M, N \text{ and } U$. [4]

   ii) List the elements of the set $M \cap N' \text{ where } N' \text{ is the complement of the set } N$. [1]

   Total 5 marks

2. Factorise the following completely:

   a) $5ab^2 + 15a^2b$ [2]

   b) $16x^2 - 81z^2$ [2]

   c) $6x^2 - 9x - 2xy + 3y$ [3]

   Total 7 marks

3. Solve the following equations:

   a) $6x^2 + 4 = 11x$ [3]

   b) $(x + 1)(2x + 1) = 15$ [4]

   c) $4x^2 + x - 7 = 0$ correct to 2 decimal places [4]

   Total 11 marks

4. Given that $f(x) = 2 - 4x - x^2$

   i) Write $f(x)$ in the form $f(x) = k - a(x + h)^2, where\ a, h \text{ and } k\ \in \mathbb{R}$. [3]

   ii) State the value of $x$ at which the maximum occurs . [1]

   iii) Determine the maximum value of the function. [1]

   Total 5 marks

5. Solve the pair of simultaneous equations

$$x - y = 1$$
$$2x^2 - y^2 = 14$$

   Total 6 marks

6. Make $r$ the subject of the following formulae:

   $a)\ p = \dfrac{5}{3 + r}$ [3]

   $b)\ M = \dfrac{L - r^2}{2N}$ [3]

   $c)\ s = \sqrt{\dfrac{r}{r + t}}$ [5]

   Total 11 marks

7. a) Using a scale of 1 cm to represent 1 unit on BOTH the $x \text{ and } y$ axes, draw on graph paper the triangles $PQR \text{ and } P'Q'R'$ such that

   $P(-4, -2), Q(-2, -2), R(-2, -5) \text{ and } P'(3, 4), Q'(5, 4) \text{ and } R'(5, 1)$. [3]

   b) FULLY describe the transformation that maps triangle $PQR$ onto triangle $P'Q'R'$. [3]

   Total 6 marks

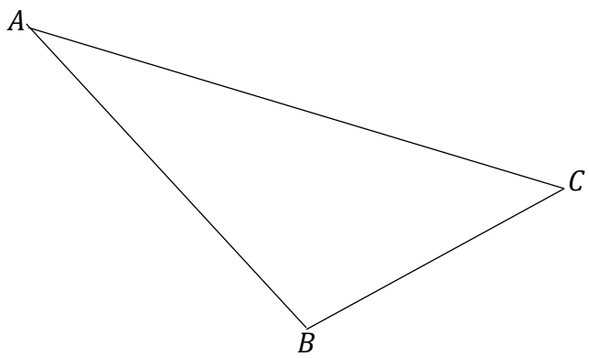
8. A boat leaves a dock at point A and travels for a distance of 15 km to point B on a bearing of $135^0$. The boat then changes course and travels for a distance of 8 km to point C on a bearing of $060^0$.

   a) Copy the diagram below, (not drawn to scale), and use it to illustrate the above information. The diagram should be clearly labelled to show the

      i) North direction [2]

      ii) Bearings $135^0$ $and$ $060^0$ [2]

      iii) Distances 8 $km$ $and$ 15 $km$ [2]




   b) Calculate

      i) The distance $AC$ [3]

      ii) The angle $BCA$ [3]

      iii) The bearing of $A$ $from$ $C$ [2]

      iv) The area of triangle $ABC$ [2]

Total 16 marks

9. The vertices of a quadrilateral, $OABC$, are $(0,0), (4,2), (6,10)$ $and$ $(2,8)$ respectively. Use a **vector method** to answer the following questions.

   a) Write as column vectors

      i) $\overrightarrow{OA}$ [1]

      ii) $\overrightarrow{CB}$ [2]

      iii) $\overrightarrow{OM}$ where $M$ is the mid-point of the diagonal $AC$. [4]

   b) Calculate the **EXACT** length of the vector $\overrightarrow{OA}$. [1]

   c) State two geometrical relationships between the line segments $OA$ $and$ $CB$. [2]

Total 10 marks

10. Given the linear equations

$$2x + y = 2$$

$$7x + 4y = 3$$

i) Write the equations in the form $AX = B$ $where$ $A, X$ $and$ $B$ are matrices. [2]

ii) Calculate the determinant of the matrix $A$. [1]

iii) Find the inverse of the matrix $A, A^{-1}$. [2]

iv) Use the matrix $A^{-1}$ to solve the linear equations for $x$ $and$ $y$. [3]

Total 8 marks

END OF TEST