HARRISON COLLEGE

END OF YEAR EXAMINATION 2019

FOURTH YEAR MATHEMATICS

PAPER 02

DURATION: 2 Hours

INSTRUCTIONS TO CANDIDATES

1) This question paper consists of **SEVEN** printed pages and 10 questions.
2) Write your name clearly on **EACH** sheet of paper used.
3) All questions are to be attempted.
4) All working must be clearly shown.
5) Number your responses carefully and identically (including any associated parts) as they appear on the question paper.

   DO NOT WRITE ANY responses beside each other.
6) Calculators are allowed.
7) If a numerical answer cannot be given **exactly**, and the accuracy required is not specified in the question, then in the case of an angle, it must be given correct to one (1) decimal, and in others cases it must be given correct to three (3) significant figures.
8) The maximum mark for this examination is 80.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

CSEC Mathematics
LIST OF FORMULAE

Volume of Prism $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of Cylinder $V = \pi r^2 h$ where r is the radius of the base and h is the perpendicular height.

Volume of a right pyramid $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference $C = 2\pi r$ where r is the radius of the circle.

Arc length $S = \frac{\theta}{360} \times 2\pi r$ where θ is the angle subtended by the arc, measured in degrees.

Area of a circle $A = \pi r^2$ where r is the radius of the circle.

Area of a sector $A = \frac{\theta}{360} \times \pi r^2$ where θ is the angle of the sector, measured in degrees.

Area of Trapezium $A = \frac{1}{2}(a+b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

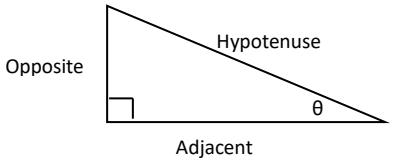
Roots of quadratic equations If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

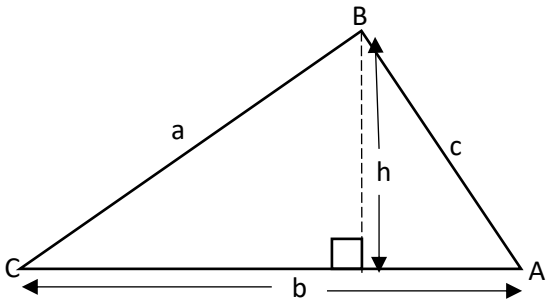
$\sin\theta = \frac{opposite \quad side}{hypotenuse}$

$\cos\theta = \frac{adjacent \quad side}{hypotenuse}$

$\tan\theta = \frac{opposite \quad side}{adjacent \quad side}$



Area of a triangle Area of $\Delta = \frac{1}{2}bh$ where *b* is the length of the base and *h* is the perpendicular height



Area of $\Delta ABC = \frac{1}{2}ab\sin C$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

1. Factorise the following completely:

   a) $3 - 12x^2$ [2]

   b) $2x^2 - 7x - 15$ [2]

   c) $3ab - b + 6a - 2$ [2]

   Total 6 marks

2. Solve the following equations:

   a) $3x^2 - 5x + 6 = 10 - 4x$ [3]

   b) $(x - 1)^2 = 21 - x$ [4]

   c) $2x^2 - 5x + 1 = 0$ correct to 2 decimal places [4]

   Total 11 marks

3. Given that $f(x) = 2x^2 + 6x - 9$

   i) Write $f(x)$ in the form $f(x) = a(x + h)^2 + k, where\ a, h\ and\ k \in \mathbb{R}$. [3]

   ii) State the value of x at which the minimum occurs. [1]

   iii) Determine the minimum value of the function. [1]

   Total 5 marks

4. Solve the pair of simultaneous equations

$$7x - y = -15$$
$$y = 2x^2$$

   Total 5 marks

5. Make a the subject of the following formulae:

   $a)$ $v^2 = u + 2a^2s$ [3]

   $b)$ $M = \frac{a+R}{3a}$ [3]

   $c)$ $s = \sqrt{\frac{a}{r}} + t$ [3]

Total 9 marks

6. (a) Given $f(x) = x^3$ and $g(x) = \frac{2x-1}{3}$,

   Calculate:

   (i) $f(-2)$ [1]

   (ii) $gf(2)$ [2]

   (iii) $g^{-1}(x)$ [3]

Total 6 marks

7. The transformation, M, denotes a reflection in the line $x = 2$. The transformation, T, denotes a translation represented by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$.

   (i) On graph paper, using a scale of 2cm to 1 unit on both the *x* and *y* axes, plot the points $A(4,2)$ and $B(3,1)$. Join AB. [2]

   On your graph:

   (ii) Draw the image $A'B'$ of AB under the transformation, M. [2]

   (iii) Draw the image $A''B''$ of AB under the transformation, T. [2]

   (iv) Describe, geometrically, a single transformation which will map $A''B''$ onto $A'B'$. [2]

Total 8 marks

8. Q, R and S are three points on level ground. R is on a bearing of 135° from Q and QR = 50 metres. The bearing of S from R is 060^0 and SR = 25 metres .

(i) Draw a diagram to show this information indicating clearly the north direction, bearings and distances given. [4]

(ii) Calculate the distance QS. [3]

(iii) Calculate, the angle QSR [3]

(iv) Calculate the bearing of Q from S. [2]

Total 12 marks

9. (a) Given that $S = \begin{pmatrix} x & 9 \\ -2 & 3 \end{pmatrix}$ is a singular matrix, determine the value of x. [2]

(b) If $A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 1 & -1 \\ 3 & 3 \end{pmatrix}$ determine:

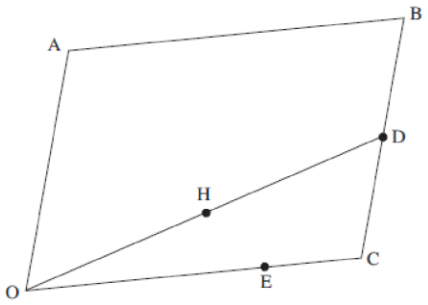
(i) BA [2]

(ii) A^2 [2]

(iii) A^{-1} [2]

Total 8 marks

10.



OABC is a parallelogram.

D is the midpoint of CB.

H is the midpoint of OD.

$\overrightarrow{OE} = \frac{2}{3}\overrightarrow{OC}$

Vector $\overrightarrow{OA} = \mathbf{a}$ and vector $\overrightarrow{OC} = \boldsymbol{c}$

(a) Express, in terms of **a** and **c**:

i) $\overrightarrow{OD}$ [2]

ii) $\overrightarrow{AE}$ [2]

iii) $\overrightarrow{HE}$. [2]

iv) State two geometrical relationships between the line segments AH and HE. [2]

(b) If $\overrightarrow{OP} = \begin{pmatrix}2\\1\end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix}3\\4\end{pmatrix}$ are the position vectors of P and Q respectively, determine

(i) $\overrightarrow{PQ}$ in the form $\begin{pmatrix}a\\b\end{pmatrix}$. [1]

(ii) $\left|\overrightarrow{PQ}\right|$ [1]

Total 10 marks

END OF TEST