FOURTH FORMS MATHEMATICS P.ST. HILL

2011 160 copies

Time: 2 hours

INSTRUCTIONS

Write your name clearly on each sheet of paper used.

Answer all questions. There are 9 questions for a total of 70 marks.

Number your answers carefully and do NOT do questions beside one another.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to 2 decimal places.

Formulae and graph paper are provided. Electronic calculators may be used.

The number of marks available is shown in the [ ] at the end of each question.

LIST OF FORMULAE

Roots of quadratic equations If $ax^2+bx+c=0$

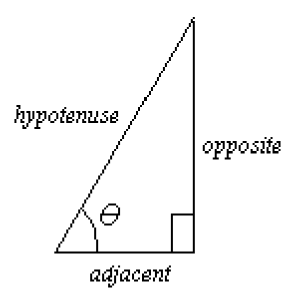
then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

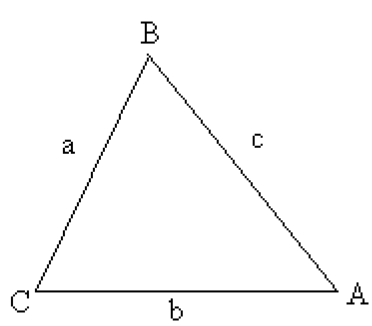
Trigonometric ratios

$\sin\theta=\dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta=\dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta=\dfrac{\text{opposite side}}{\text{adjacent side}}$





Sine rule $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

Cosine rule $a^2=b^2+c^2-2bc\cos A$

Area of triangle $\text{Area of }\triangle ABC=\frac{1}{2}ab\sin C$

1. In a survey, 65 visitors were asked which attraction they visited. Every visitor visted at least one of three attractions: the Barbados Museum, Harrison's Cave and the Atlantis Submarine.

   39 visited Harrison's Cave (H)
   25 visted the Barbados Museum (M)
   38 went on the Atlantis Submarine(A)
   9 visted the Barbados Museum and Harrison's cave
   13 visted the Barbados Museum and Atlantis Submarine
   $3x$ visited both Harrison's Cave and Atlantis Submarine ONLY
   x visited all three attractions.

   (a) Draw a Venn diagram to shows the above information, showing in terms of x the number of visitors in each subset. [7]

   (b) Determine the value of x. [x = 5] [2]

2. (a) Make w the subject of the formula $k = \sqrt{3w + a}$ $\quad w = \frac{k^2 - a}{3}$ [3]

   (b) (i) Factorise $2m - 5x - xm + 10$ $\quad = (2 - x)(m + 5)$ [3]

   (ii) Simplify after factorising the numerator and denominator: $\frac{x^2 - 4}{2x^2 - x - 6}$ [3]

   $$= \frac{(x+2)}{(2x+3)}$$

3. Solve the simultaneous equations $x^2 + y^2 = 17$

   $y = x - 3$ [7]

   Answer : $(-1, -4)$ and $(4, 1)$

4. Given $f(x) = x^2 - 16x + 4$

   (i) Write $f(x)$ in the form $a(x-h)^2 + k$ where a, h and $k \in \mathbb{R}$. [3]

   (ii) Write down the coordinates of the minimum point of the graph of $y = f(x)$. [2]

   (iii) Solve giving your answer correct to 2 decimal places:

   $x^2 - 16x + 4 = 0$ $\quad f(x) = (x-8)^2 - 60$ [4]

   $x = 8 \quad y = -60$

   $x = 0.26 \quad \boxed{x = 15.75}$

5. (a) A straight line, l_1, passes through the point A(1, 1) and B($6, -2$).
   Find the gradient of the line, l_1. m= -0.6 [2]

   (b) Another line, l_2, passes through the point C($-1, 9$). l_2 is perpendicular to l_1.

Determine the equation of the line l_2. $(y-9)=\frac{5}{3}(x+1)$ [3]

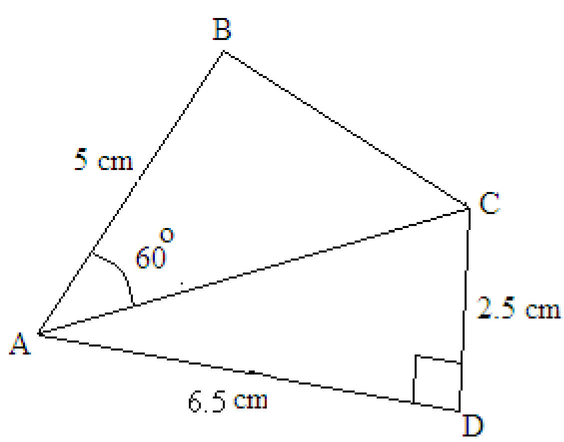
6. The functions f and g are such that

$f(x) = \frac{5x-2}{x}$ and $g(x) = x^2$

(a) Determine an expression in x for $f^{-1}(x)$ $y = f^{-1}(x) = \frac{-2}{x-5}$ or $\frac{2}{5-x}$ [5]

(b) Evaluate $gf(2)$ $= g(4) = 16$. [2]

7.



In the figure above, not drawn to scale, AB = 5 cm, AD = 6.5 cm and CD = 2.5 cm.

$\angle BAC = 60^0$, $\angle ADC = 90^0$

Calculate the size of (i) the length of AC $AC = 6.96$ [2]

(ii) the length of BC $BC = 6.22$ [3]

(iii) the size of angle ABC $angle\ ABC = 75.7^o$ [3]

8. If the matrix P = $\begin{pmatrix} 5 & -1 \\ 3 & 4 \end{pmatrix}$, and the matrix Q = $\begin{pmatrix} 7 & 8 \\ -2 & 6 \end{pmatrix}$, evaluate

(i) 2P − Q $= \begin{bmatrix} 3 & -10 \\ 8 & -2 \end{bmatrix}$ [2]

(ii) PQ $= \begin{bmatrix} 37 & 34 \\ 13 & 48 \end{bmatrix}$ [4]

[Please turn over for question 9]

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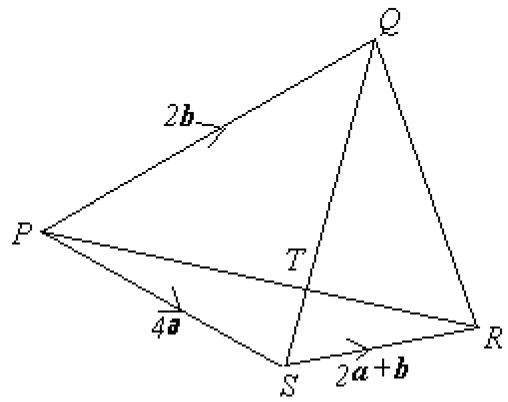

In the diagram, $\overrightarrow{PQ} = 2\boldsymbol{b}$, $\overrightarrow{PS} = 4\boldsymbol{a}$ and $\overrightarrow{SR} = 2\boldsymbol{a} + \boldsymbol{b}$.

(a) Express in terms of $\boldsymbol{a}$ and/or $\boldsymbol{b}$, simplifying your answers where possible.

(i) $\overrightarrow{SQ}$ $\quad \overline{SQ} = 2\boldsymbol{b} - 4\boldsymbol{a}$ [1]

(ii) $\overrightarrow{QR}$ $\quad = 6\boldsymbol{a} - \boldsymbol{b}$ [3]

(b) If $\overrightarrow{PT} = h\overrightarrow{PR}$, express $\overrightarrow{PT}$ in term of h, $\boldsymbol{a}$ and $\boldsymbol{b}$. $\quad = h(6\boldsymbol{a} + \boldsymbol{b})$ [2]

(c) Given that $4\overrightarrow{ST} = \overrightarrow{SQ}$, calculate the value of h. [4]

Solution to part(c): $\overline{ST} = \frac{1}{4}\overline{SQ} = \frac{1}{4}(2\boldsymbol{b} - 4\boldsymbol{a}) = \frac{1}{2}\boldsymbol{b} - \boldsymbol{a}$

$\boxed{\overline{ST} = \overline{PT} - 4\boldsymbol{a} = h(6\boldsymbol{a} + \boldsymbol{b}) - 4\boldsymbol{a}} = (6h - 4)\boldsymbol{a} + h\boldsymbol{b}$

$\boxed{(6h - 4)\boldsymbol{a} + h\boldsymbol{b}} = \frac{1}{2}\boldsymbol{b} - \boldsymbol{a}$

$h = \frac{1}{2}$