HARRISON COLLEGE

THIRD FORM MATHEMATICS

INTERNAL PROMOTION EXAMINATION 2013 - 2014

DURATION: 1 hour and 45 minutes

GENERAL INSTRUCTIONS TO CANDIDATES:

1) This question paper consists of FIVE printed pages.

2) Write your name clearly on **EACH** sheet of paper used.

3) All twenty-one questions are to be attempted.

4) Number your responses carefully and identically (including any associated parts) as they appear on the question paper.
Do **NOT** write ANY of your responses beside each other.

5) Calculators are allowed.

6) If a numerical answer cannot be given **exactly**, and the accuracy required is not specified in the question, then in the case of an angle it must be given correct to **one (1)** decimal place, in other cases it must be given correct to **three (3)** significant figures.

7) The maximum mark for this examination is 91.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

Write on your foolscap the **LETTER** that matches your response for Questions 1 – 10.

1) The mode of the sample 6, 5, 11, 9, 9, 5, 8, 5 is:

(A) 5 **(B)** 8 **(C)** 9 **(D)** 11

2) A bag contains 3 red marbles, 5 blue marbles, 5 green and 7 yellow marbles. A marble is drawn at random from the bag. The probability that this marble is green is:

(A) $\frac{5}{13}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{17}$ **(D)** $\frac{1}{4}$

3) If x rulers cost \$$p$ each and y erasers cost \$$q$ each, then the total cost of x rulers and y erasers, in dollars, is:

(A) $\frac{x}{p}+\frac{y}{q}$ **(B)** $\frac{xy}{pq}$ **(C)** $px + qy$ **(D)** $p + x + q + y$

4) $\frac{m^2n}{v^3}\times\frac{u^3v}{mn^2}$, simplifies to:

(A) $\frac{m^3n^3u^3}{v^3}$ **(B)** $\frac{mu^3}{nv^2}$ **(C)** $\frac{m^3u}{n^2v}$ **(D)** $\frac{m^6nuv}{mnv^6}$

5) $\left(\frac{ab^2}{4}\right)^3$ can be expressed as:

(A) $\frac{3ab^6}{12}$ **(B)** $\frac{a^3b^6}{12}$ **(C)** $\frac{ab^6}{64}$ **(D)** $\frac{a^3b^6}{64}$

6) Given that $x^2 + y^2 = r^2$ and $r > x$, then y =

(A) $\pm\sqrt{r^2-x^2}$ **(B)** $r - x$ **(C)** $r^2 - x$ **(D)** $\pm\frac{r^2}{x}$

7) Given that f: $x \rightarrow 3x - 5$, then $f(-2)$ =

(A) – 1 **(B)** – 11 **(C)** – 7 **(D)** – 2

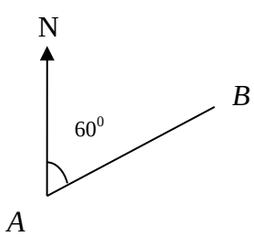
8) Given that y is directly proportional to z and k is a constant then:

(A) $y=\frac{k}{z}$ **(B)** $y=\frac{z}{k}$ **(C)** $y = kz$ **(D)** $k = yz$

9) The gradient of the line perpendicular to $2y = 3x + 5$ is:

(A) $\frac{3}{2}$ **(B)** $-\frac{2}{3}$ **(C)** 6 **(D)** $\frac{2}{3}$

10) In the diagram below, the bearing of A from B is:



(A) 240^0 **(B)** 60^0 **(C)** 300^0 **(D)** 120^0 [Total: **10**]

All working MUST be shown for questions 11 – 20

11) A plot of land is sold for $ 40 000. It appreciates in value at the rate of 2.5 % per annum. Calculate

(i) the amount of appreciation after one year. [**1**]

(ii) the value of the land after three years. [**3**]

12) y is inversely proportional to the cube of x and $y = 9$ when $x = 2$.
Calculate

(i) the value of the constant of proportionality [**2**]

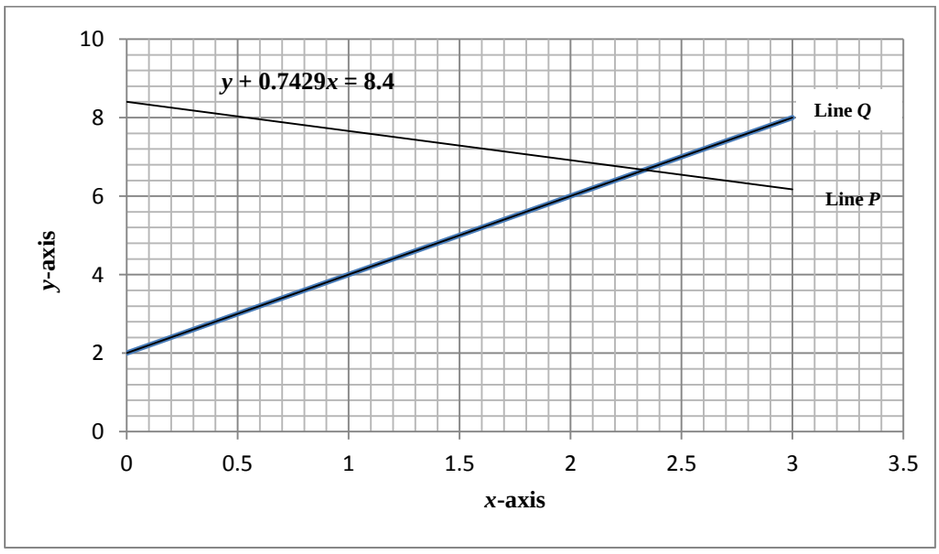
(ii) the EXACT value of y when $x = 3$. [**2**]

13) (i) Solve for x, $(x + \frac{1}{4}) \leq \frac{2}{3}(x - 1)$. [**5**]

(ii) Given that $x \in \boldsymbol{Z}$, state the largest value of x which satisfies the inequality in part (i). [**1**]

14) Solve for x and y, the simultaneous equations: $\begin{aligned} 2x &= 10 + 5y \\ 5x - 3y &= 6 \end{aligned}$ [**6**]

15) The figure below shows two lines ***P*** and ***Q***. The equation of Line ***P*** is $y + 0.7429x = 8.4$ and the equation of Line ***Q*** is of the form $y = tx + v$, where t and v are real numbers.



(i) Find the gradient of

(a) line ***P*** [**2**]

(b) line ***Q***. [**2**]

(ii) Find the equation of line ***Q***. [**2**]

(iii) Estimate the value of x and the value of y which satisfies BOTH lines ***P*** and ***Q***. [**2**]

16) Isaac is married with three children. In the year 2013, he earned $15 200.

Tax-Free Allowances per year
Personal Allowance: $1 500
Spouse: $1 000
Child: $500
National Insurance Contribution: $1 200
Tax Rates
First $3 000: 10%
Next $4 000: 20%
Next $5 000: 30%
Over $12 000: 40%

Using the information in the above table, for income year 2013, calculate

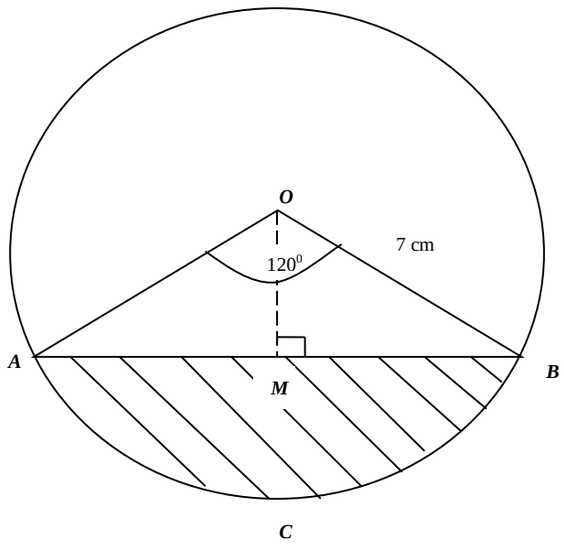
(i) his total tax-free allowances [**5**]
(ii) his taxable income [**2**]
(iii) the amount of income tax paid [**4**]
(iv) his net income. [**2**]

17) The masses of a sample of breadfruits were recorded as shown in the table below.

Mass (g)	Number of Breadfruits
1440 – 1444	45
1445 – 1449	71
1450 – 1454	12
1455 – 1459	60
1460 – 1464	12

(i) State the modal class. [**1**]
(ii) Calculate, in kilograms, the mean mass of the breadfruits. [**8**]
(iii) Determine the probability that a breadfruit chosen at random is at most 1454.5 g. [**2**]

18) In the diagram below, (not drawn to scale), *O* is the centre of the circle of radius 7 cm. Chord *AB* subtends an angle *AOB* of 120^0 at the centre of the circle.



(a) Calculate the length of *OM*. [**3**]

(b) Using the value 3.142 for π calculate

(i) the area of the sector *AOBC*. [**2**]
(ii) the area of the minor segment of the circle. [**4**]

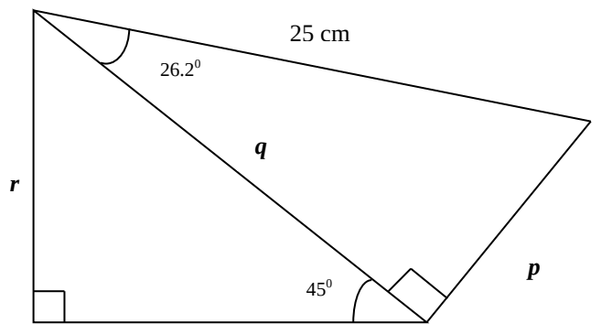
(c) Triangle *AOB* represents the opening of a hollow section of a cylindrical solid of length 20 cm formed by the circle. Calculate the volume of the material that remains in the solid.[**3**]

19) [Use the figure below to answer this question]

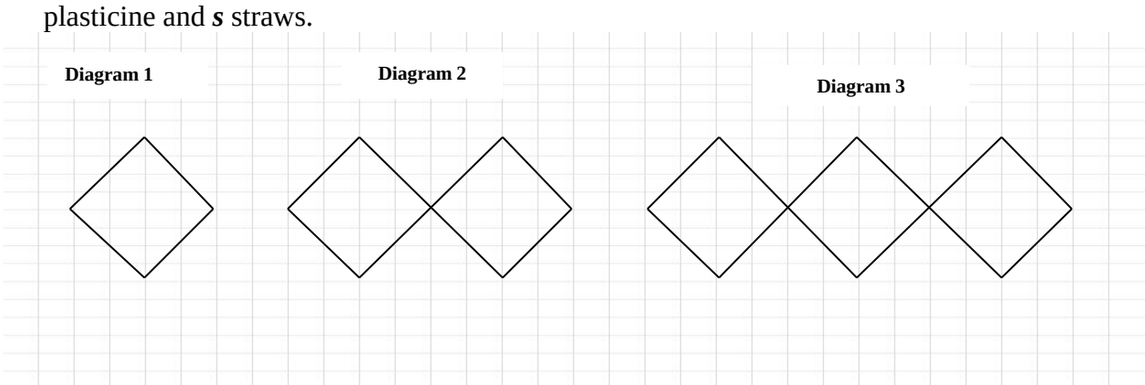
(i) Calculate the value of ***p*** [**3**]

(ii) Calculate the value of ***q*** [**3**]

(iii) Show that $\boldsymbol{r} = 25 \cos 35^0 \sin 45^0$ cm. [**3**]



20) The Figure below shows the first three diagrams in a sequence. Each diagram is made up of straws joined together by a piece of plasticine at each vertex (or corner). The straws are represented by lines and the plasticine by dots. In each diagram, there are ***p*** pieces of plasticine and ***s*** straws.



(a) (i) Determine how many straws would be in the FOURTH diagram [**1**]

(ii) Determine how many pieces of plasticine would be in the FIFTH diagram. [**1**]

(b) COPY and COMPLETE the table below by inserting the missing values at the rows marked (i) a) and (i) b); (ii) a) and (ii) b). [**4**]

No. of Straws, *s*	**Rule Connecting *p* and *s***	**No. of Pieces of Plasticine, *p***
4	$1 + \left(\frac{3}{4} \times 4\right)$	4
8	$1 + \left(\frac{3}{4} \times 8\right)$	7
12	$1 + \left(\frac{3}{4} \times 12\right)$	10
(i) 52	a) _____	b) ___
(ii) a) ___	b) _____	55

(c) Write the rule, in terms of ***p*** and ***s***, to show how ***p*** is related to ***s***. [**2**]

End of Examination

Solutions and Marking Key – Third Form Mathematics 2013 – 2014

1) (A) **2) (D)** **3) (C)** **4) (B)** **5) (D)**

6) (A) **7) (B)** **8) (C)** **9) (B)** **10) (A)**

11) P = \$ 40 000, R = 2.5 %

(i) $\frac{2.5}{100} \times$ \$ 40 000 = \$ 1000 OR ($\frac{102.5}{100} \times$ \$ 40 000) – \$ 40 000 = \$ 1000

(ii) Value = $P(1 + \frac{R}{100})^n$ OR After 1 year = \$ 41 000

$= 40\,000(1 + \frac{2.5}{100})^3$ After 2 years = \$ 41 000 × 1.025

= \$ 43 075.625 = \$ 42 025

After 3 years = \$ 42 025 × 1.025

= \$ 43 075.625

12) $y \ \alpha \frac{1}{x^3} \rightarrow y = \frac{k}{x^3}$

(i) $x = 2, y = 9 \rightarrow 9 = \frac{k}{2^3}$

$72 = k$

(ii) $y = \frac{72}{x^3}$

$x = 3$: $y = \frac{72}{3^3} = \frac{8}{3}$

13) (i) $(x + \frac{1}{4}) \leq \frac{2}{3}(x - 1)$

$12(x + \frac{1}{4}) \leq 12[\frac{2}{3}(x - 1)]$

$12x + 3 \leq 8(x - 1)$

$12x + 3 \leq 8x - 8$

$4x \leq -11$

$x \leq -\frac{11}{4}$

(ii) $x \leq -\frac{11}{4}, x \in \boldsymbol{Z}$

$\therefore x = -3$

14) $2x = 10 + 5y \quad Eqn\ (1)$
$5x - 3y = 6 \quad Eqn\ (2)$

From (1) $x = \frac{10+5y}{2}$

Sub. into (2): $5(\frac{10+5y}{2}) - 3y = 6$

$5(10 + 5y) - 6y = 12$

$50 + 25y - 6y = 12$

$19y = -38$

$y = -2$

Sub. into $x = \frac{10+5y}{2} = \frac{10+5(-2)}{2} = 0$

OR $2x = 10 + 5y \quad Eqn\ (1) \times 5$
$5x - 3y = 6 \quad Eqn\ (2) \times (-2)$

$10x - 25y = 50$
$-10x + 6y = -12$

$-19y = 38$

$y = -2$

Sub. into Eqn (1)

$2x = 10 + 5(-2)$

$x = 0$

15) (i) (a) Line P: $y + 0.7429x = 8.4$

$y = -0.7429x + 8.4$

Gradient $= -0.7429x$

(b) Gradient of Line Q: $\frac{8-2}{3-0}$

$= \frac{6}{3}$

$= 2$

(ii) $\frac{y-2}{x-0} = \frac{2}{1}$

$y - 2 = 2x$

(iii) $x = 2.35 \pm 0.1$, $y = 6.6 \pm 0.1$

16) (i) Total Tax-Free Allowance = \$ 1 500

+ \$ 1 000

+ \$ (600 × 3)

+ \$ 1 200

\$ 5 500

(ii) Taxable Income = \$ 15 200 – \$ 5 500

= \$ 9 700

(iii) Income Tax Paid = $(\frac{10}{100} \times \$3\,000) + (\frac{20}{100} \times \$4\,000) + (\frac{30}{100} \times \$2\,700)$

= \$ 300 + \$ 800 + \$ 810

= \$ 1 900

(iv) Net Income = \$ 15 200 – \$ 1 900

= \$ 13 290

17)

Mass (g)	Number of Breadfruits, *f*	Mid-Interval Value
1440 – 1444	45	$\frac{1439.5+1444.5}{2} = 1442$
1445 – 1449	71	$\frac{1444.5+1449.5}{2} = 1447$
1450 – 1454	12	$\frac{1449.5+1454.5}{2} = 1452$
1455 – 1459	60	$\frac{1454.5+1459.5}{2} = 1457$
1460 – 1464	12	$\frac{1459.5+1464.5}{2} = 1462$

(i) The modal class is (1445 – 1449)g

(ii) The mean mass $= \frac{(45\times1442) + (71\times1447)+(12\times1452)+(60\times1457)(45\times1462)}{45 + 71 + 12+60 + 12}$

$= \frac{64\,890 + 102\,737+ 17\,424 + 87\,420 + 17\,544}{45 + 71 + 12+60 + 12}$

$= \frac{290\,015}{200}$

= 1450. 075 g

= 1.450075 kg

= 1.45 kg (3 s.f.)

(iii) P (x is at most 1454.5 g) i.e. P ($x \leq 1454.5$ g) $= \frac{45 + 71 + 12}{200}$

$= \frac{16}{25}$

18) (a) $\cos 60^0 = \frac{OM}{7cm}$ OR $\sin 30^0 = \frac{OM}{7cm}$

$\cos 60^0 \times 7\ cm = OM$ OR $\sin 30^0 \times 7\ cm = OM$

$3.5\ cm = OM$ OR $3.5\ cm = OM$

(b) (i) Area of sector $AOBC = \frac{120}{360} \times \pi r^2$

$= \frac{1}{3} \times 3.142\ \times (7cm)^2$

$= 51.3\ cm^2$ (3 s.f.)

(ii) Now $\cos 30^0 = \frac{MB}{7cm}$

$6.06\ cm$ (3 s.f.) $= MB$

So Area of Minor Segment $= 51.3\ cm^2 - [(\frac{1}{2} \times 6.06cm \times 3.5cm) \times 2]$

$= 30.1\ cm^2$ (3 s.f.)

(c) Volume of solid $= (3.142 \times 7^2 \times 20) - (6.06 \times 3.5\ \times 20)$

$= 2654.96\ \text{cm}^3$

19) (i) $\sin 35^0 = \frac{p}{25cm}$

$\sin 35^0 \times 25\ cm = p$

$14.3\ cm$ (3 s.f.) $= p$

(ii) $\cos 35^0 = \frac{q}{25cm}$

$\cos 35^0 \times 25\ cm = q$

$20.5\ cm$ (3 s.f.) $= q$

(iii) $\sin 45^0 = \frac{r}{q}$

$\sin 45^0 = \frac{r}{25cos35^0 cm}$

$25 \cos 35^0 \sin 45^0\ \text{cm} = r.$

20) (a) (i) 16 straws

(ii) 16 pieces of plasticine

(b) (i) a) $1 + \left(\frac{3}{4} \times 52\right)$ b) 40

(ii) a) 72 b) $1 + \left(\frac{3}{4} \times 72\right)$

(c) $\boldsymbol{p} = 1 + \left(\frac{3}{4} \times s\right)$ OR $4\boldsymbol{p} = 4 + 3\boldsymbol{s}$