TEST CODE **01254020**

**FORM TP 2019040**

MAY/JUNE 2019

# CARIBBEAN EXAMINATIONS COUNCIL

## CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
## EXAMINATION

## ADDITIONAL MATHEMATICS

**Paper 02 – General Proficiency**

***2 hours 40 minutes***

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**READ THE FOLLOWING INSTRUCTIONS CAREFULLY.**

1. This paper consists of FOUR sections. Answer ALL questions in Section I, Section II and Section III.
2. Answer ONE question in Section IV.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. A list of formulae is provided on page 4 of this booklet.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

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**<u>Required Examination Materials</u>**

Electronic calculator (non-programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

**DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.**

**LIST OF FORMULAE**

| | |
|---|---|
| Arithmetic Series | $T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$ |
| Geometric Series | $T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r};\ -1 < r < 1 \text{ or } \lvert r \rvert < 1$ |
| Circle | $x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x+f)^2 + (y+g)^2 = r^2$ |
| Vectors | $\hat{\mathbf{v}} = \frac{\mathbf{v}}{\lvert\mathbf{v}\rvert} \qquad \cos\theta = \frac{\mathbf{a}\bullet\mathbf{b}}{\lvert\mathbf{a}\rvert \times \lvert\mathbf{b}\rvert} \qquad \lvert\mathbf{v}\rvert = \sqrt{(x^2 + y^2)}$ where $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$ |
| Trigonometry | $\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$ <br> $\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$ <br> $\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ |
| Differentiation | $\frac{\mathrm{d}}{\mathrm{d}x}(ax+b)^n = an(ax+b)^{n-1}$ <br> $\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$ <br> $\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$ |
| Statistics | $\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\overline{x})^2$ |
| Probability | $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ |
| Kinematics | $v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$ |

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## SECTION I

**Answer BOTH questions.**

**ALL working must be clearly shown.**

**1.** (a) The function $f$ is such that $f(x) = 2x^3 + 7x^2 + 3x$.

(i) Determine **all** linear factors of $f(x)$.

**(3 marks)**

(ii) Compute the roots of the function $f(x)$.

**(2 marks)**

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(b) Two functions are such that $g(x) = x^2 - x$ and $h(x) = 2x - 3$.

(i) Determine $gh(x)$.

**(2 marks)**

(ii) Given that $hg(x) = 2x^2 - 2x - 3$, show that the values of $x$, for which $hg(x) = 0$, can be expressed as $\frac{1 \pm \sqrt{7}}{2}$.

**(3 marks)**

(c) Solve $3x \log 2 + \log 8^x = 2$.

**(4 marks)**

**Total 14 marks**

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**2.** (a) (i) Express $f(x) = -2x^2 - 7x - 6$ in the form $a(x + h)^2 + k$.

**(3 marks)**

(ii) State the maximum value of $f(x)$.

**(1 mark)**

(iii) State the value of $x$ for which $f(x)$ is a maximum.

**(1 mark)**

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(iv) Use your answer in (a) (i) to determine **all** values of $x$ when $f(x) = 0$.

**(3 marks)**

(v) Sketch the function $f(x)$ and show your solution set to (a) (iv) when $f(x) < 0$.

**(2 marks)**

(b) A geometric series can be represented by $\frac{y}{x} + \frac{y^2}{x^3} + \frac{y^3}{x^5} + \ldots$

Prove that $S_\infty = xy\,(x^2 - y)^{-1}$ .

**(4 marks)**

**Total 14 marks**

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## SECTION II

**Answer BOTH questions.**

**ALL working must be clearly shown.**

**3.** (a) A circle with centre (1, –1) passes through the point (4, 3).

(i) Calculate the radius of the circle.

**(2 marks)**

(ii) Write the equation of the circle in the form $x^2 + y^2 + 2fx + 2gy + c = 0$.

**(2 marks)**

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(iii) Determine the equation of the tangent to the circle at the point (4, 3).

**(3 marks)**

(b) Two vectors **p** and **q** are such that **p** = 8**i** + 2**j** and **q** = **i** – 4**j**.

(i) Calculate **p.q**.

**(2 marks)**

(ii) State the angle between the two vectors **p** and **q**.

**(1 mark)**



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(c) The position vector $\mathbf{a} = 4\mathbf{i} - 7\mathbf{j}$. Find the unit vector in the direction of $\mathbf{a}$.

**(2 marks)**

**Total 12 marks**

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**4.** (a) A compass is used to draw a sector of radius 6 cm and area 11.32 $cm^2$.

(i) Determine the angle of the sector in radians.

**(3 marks)**

(ii) Calculate the perimeter of the sector.

**(2 marks)**

(b) A right-angled triangle **XYZ** has an angle, $\theta$, where $\sin\theta = \frac{\sqrt{5}}{5}$.

Without evaluating $\theta$, calculate the exact value (in surd form if applicable) of

(i) $\cos\theta$

**(2 marks)**

(ii) $\sin 2\theta$.

**(2 marks)**

(c) Show that $\tan^2\theta + 1 = \dfrac{1}{\cos^2\theta}$.

**(3 marks)**

**Total 12 marks**

## SECTION III

**Answer BOTH questions.**

**ALL working must be clearly shown.**

**5.** (a) The stationary points of a curve are given by $(5, 11\frac{2}{3})$ and $(3, 15)$.

(i) Derive an expression for $\frac{dy}{dx}$.

**(2 marks)**

(ii) Determine the nature of the stationary points.

**(5 marks)**

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(iii) Determine the equation of the curve.

**(4 marks)**

(b) Differentiate $\sqrt[3]{(2x+3)^2}$ with respect to $x$, giving your answer in its simplest form.

**(3 marks)**

**Total 14 marks**

**6.** (a) Integrate $3 \cos x + 2 \sin x$ .

**(2 marks)**

(b) Evaluate $\int_1^4 \frac{2\sqrt{x}}{x}\, dx$ .

**(4 marks)**

(c) The point (2, 4) lies on the curve whose gradient is given by $\frac{dy}{dx} = -2x + 1$.

Determine

(i) the equation of the curve

**(4 marks)**

(ii) the area under the curve in the finite region in the first quadrant between 0 and 3 on the $x$-axis.

**(4 marks)**
**Total 14 marks**

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## SECTION IV

**Answer only ONE question.**

**ALL working must be clearly shown.**

**7.** (a) The weights, in kg, of students in a Grade 5 class are displayed in the following stem and leaf diagram.

| Boys | | Girls |
|---|---|---|
| 9 8 | 2 | 8 8 9 |
| 9 9 9 7 3 | 3 | 2 2 3 5 8 8 8 |
| 5 1 1 1 1 0 0 | 4 | 0 1 1 2 |
| 1 | 5 | |

**Key:**

**Boys 8|2 means 28 kg**
**Girls 2|8 means 28 kg**

(i) State the number of students in the class.

..........................................................................................................................

**(1 mark)**

(ii) Construct ONE box-and-whisker plot for the entire Grade 5 class (boys and girls combined).

**(4 marks)**

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(iii) The standard deviation of the weights of the boys is 5.53 kg.

Determine the standard deviation of the weights of the girls. Provide an interpretation of your answer for the girls compared to that given for the boys.

Interpretation ..........................................................................................

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**(5 marks)**

(iv) Determine the number of students above the 20th percentile for this class.

**(2 marks)**

(b) A vendor has 15 apples on a tray: 5 red, 6 green and 4 yellow. A customer requests 3 apples but does NOT specify a colour.

Determine the probability that the apples chosen

(i) contain one of EACH colour

**(4 marks)**

(ii) are ALL of the same colour.

**(4 marks)**

**Total 20 marks**

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**8.** (a) A car has stopped at a traffic light. When the light turns green, it accelerates uniformly to a speed of 28 $ms^{-1}$ in 15 seconds. The car continues to travel at this speed for another 35 seconds, before it has to stop 10 seconds later at another traffic light.

(i) On the grid provided **on page 25**, draw a speed–time graph showing the information above. **(3 marks)**

(ii) Calculate the distance the car travelled between the two traffic lights.

**(3 marks)**

(iii) Calculate the average speed of the car over this journey, **giving your answer in** $\mathbf{kmh^{-1}}$.

**(3 marks)**

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(b) A particle moves in a straight line such that $t$ seconds after passing a fixed point, $O$, its acceleration, $a$, in $ms^{-2}$, is given by $a = 12t - 17$. Given that its speed at $O$ is $10\ ms^{-1}$, determine

(i) the values of $t$ for which the particle is stationary

**(5 marks)**

(ii) the distance the particle travels in the fourth second.

**(6 marks)**

**Total 20 marks**

**END OF TEST**

**IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.**