FORM TP 2018037

MAY/JUNE 2018

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of FOUR sections. Answer ALL questions in Section I, Section II and Section III.
2. Answer ONE question in Section IV.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. A list of formulae is provided on page 4 of this booklet.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Required Examination Materials

Electronic calculator (non-programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Arithmetic Series $\quad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric Series $\quad T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r}, -1 < r < 1 \text{ or } |r| < 1$

Circle $\quad x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x + f)^2 + (y + g)^2 = r^2$

Vectors $\quad \hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)}$ where $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Differentiation

$$\frac{\mathrm{d}}{\mathrm{d}x}(ax + b)^n = an(ax + b)^{n-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$$

Statistics

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\overline{x})^2$$

Probability $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Kinematics $\quad v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$

SECTION I

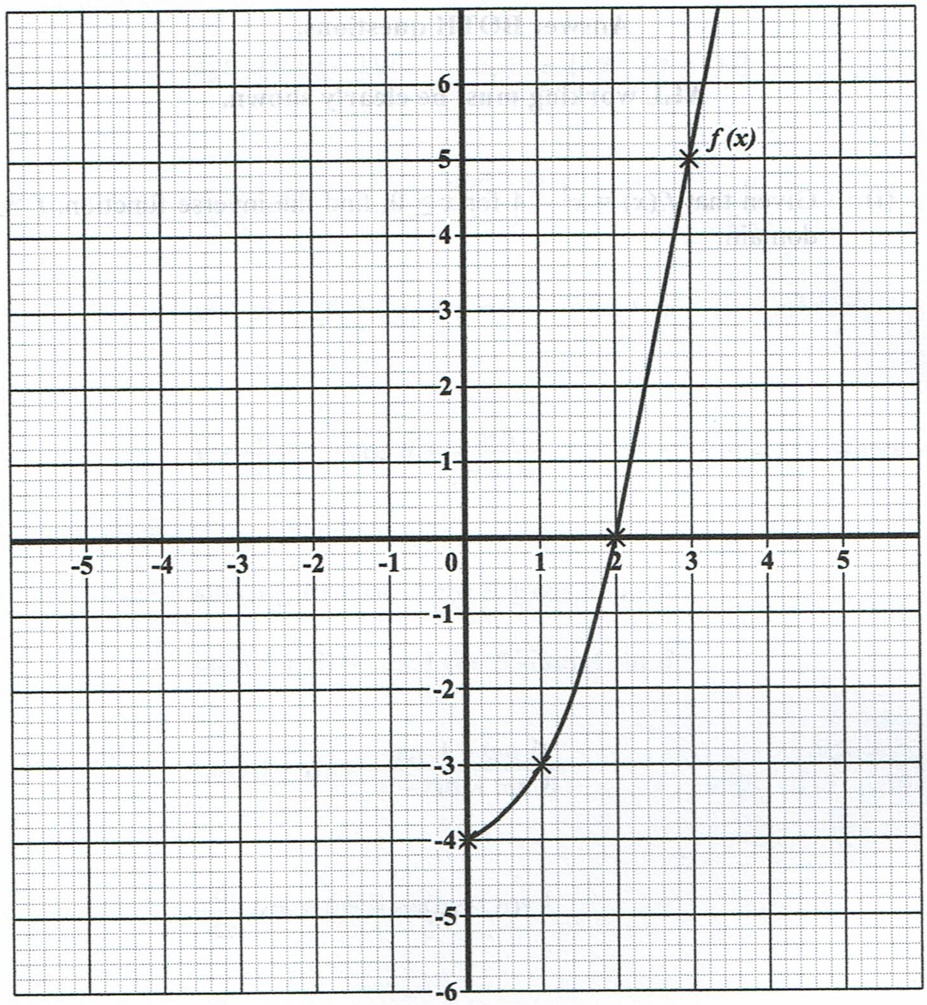
Answer BOTH questions.

ALL working must be clearly shown.

1. (a) (i) Given that $f(x) = x^2 - 4$ for $x \geq 0$, find the inverse function, $f^{-1}(x)$, stating its domain.

(4 marks)

(ii) On the grid provided below, sketch $f^{-1}(x)$.



(2 marks)

(iii) State the relationship between $f(x)$ and $f^{-1}(x)$.

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(2 marks)

(b) Derive the polynomial, $P(x)$, of degrees 3 which has roots equal to 1, 2 and -4.

(3 marks)

(c) An equation relating V and t is given by $V = ka^t$ where k and a are constants.

(i) Use logarithms to derive an equation of the form $y = mx + c$ that can be used to find the values of k and a.

(2 marks)

(ii) If a graph of y versus x from the equation in Part (c) (i) is plotted, a straight line is obtained. State an expression for the gradient of the graph.

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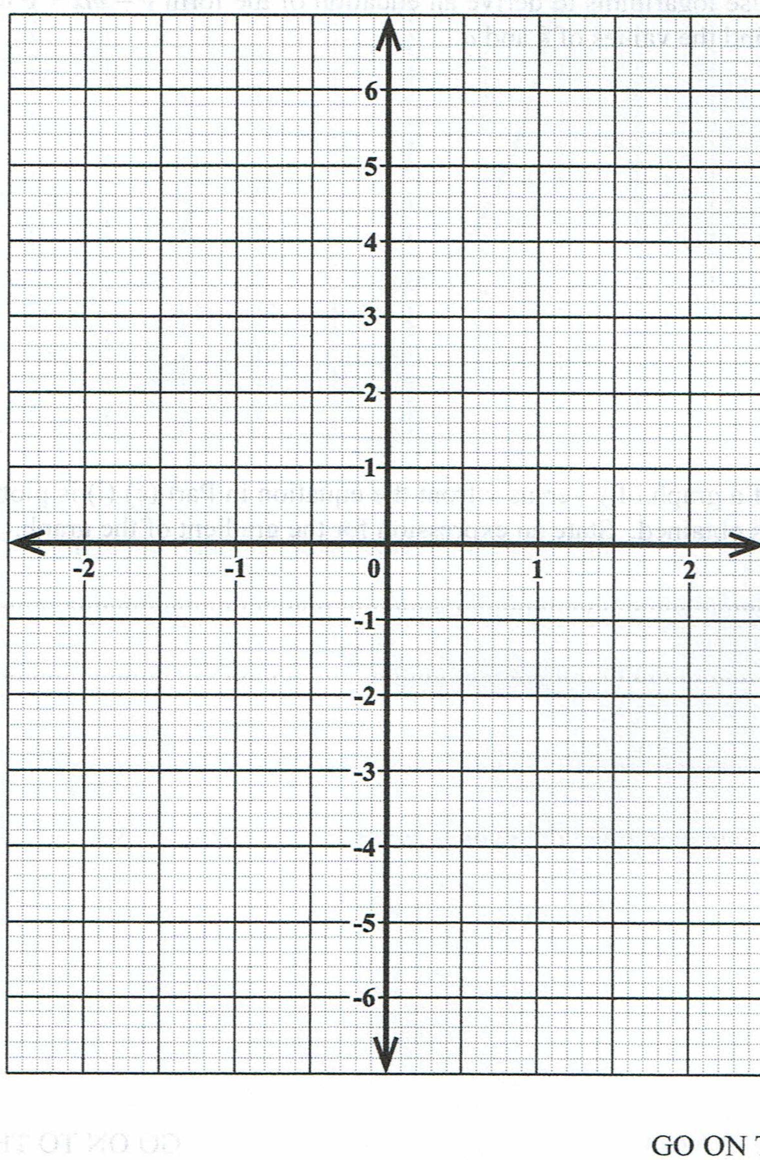
(1 mark)

Total 14 marks

2. (a) (i) Given that $g(x) = -x^2 + x - 3$, express $g(x)$ in the form $a(x + h)^2 + k$ where a, h and k are constants.

(3 marks)

(ii) On the grid provided below, sketch the graph of $g(x)$, showing the maximum point and the y-intercept.



(3 marks)

(b) In a geometric progression, the 3rd term is 25 and the sum of the 1st and 2nd terms is 150. Determine the sum of the first four terms, given that $r > 0$.

(4 marks)

(c) If α and β are the roots of the equation $2x^2 - 5x + 3 = 0$, determine the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

(4 marks)

Total 14 marks

SECTION II

Answer BOTH questions.

ALL working must be clearly shown.

3. (a) Determine the equation of the circle that has centre (5, –2), and passes through the origin.

(3 marks)

(b) Determine whether the following pair of lines is parallel.

$$x + y = 4$$
$$3x - 2y = -3$$

(2 marks)

(c) The position vectors of two points, A and B, relative to a fixed origin, O, are given by $\overrightarrow{\mathbf{OA}} = 2\mathbf{i} + \mathbf{j}$ and $\overrightarrow{\mathbf{OB}} = 3\mathbf{i} - 5\mathbf{j}$, where **i** and **j** represent the unit vectors in the x and y directions respectively. Calculate

(i) the magnitude of $\overrightarrow{\mathbf{AB}}$

(4 marks)

(ii) the angle $\mathbf{A\hat{O}B}$, giving your answer to the nearest whole number.

(3 marks)

Total 12 marks

4. (a) A wire in the form of a circle with radius 4 cm is reshaped in the form of a sector of a circle with radius 10 cm. Determine, in radians, the angle of the sector, giving your answer in terms of π.

(4 marks)

(b) Solve the equation $\sin^2\theta + 3\cos 2\theta = 2$ for $0 \leq \theta \leq \pi$. Give your answer(s) to 1 decimal place.

(4 marks)

(c) Prove the identity $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} \equiv \dfrac{2\tan x}{\cos x}$.

(4 marks)

Total 12 marks

SECTION III

Answer BOTH questions.

ALL working must be clearly shown.

5. (a) Given that $y = x^3 + 2x^2 - 1$, determine

(i) the coordinates of the stationary points

(5 marks)

(ii) the nature of EACH stationary point.

(5 marks)

(b) Differentiate $y = 2x\sqrt{(4-8x)}$ with respect to x, simplifying your answer.

(4 marks)

Total 14 marks

6. (a) Show, using integration, that the finite area of the curve $y = \sin x$ in the first quadrant bounded by the line $x = \frac{4\pi}{9}$ is smaller than the finite region of $y = \cos x$ in the same quadrant and bounded by the same line.

(6 marks)

(b) The finite region in the first quadrant bounded by the curve $y = x^2 + x + 3$, the x-axis and the line $x = 4$ is rotated completely about the x-axis. Determine the volume of the solid of revolution formed.

(4 marks)

(c) A curve which has a gradient of $\frac{dy}{dx} = 3x - 1$ passes through the point $A(4, 1)$. Find the equation of the curve.

(4 marks)

Total 14 marks

SECTION IV

Answer only ONE question.

ALL working must be clearly shown.

7. (a) The number of runs scored by a cricketer for 18 consecutive innings is illustrated in the following stem-and-leaf diagram.

0	2	3	6	7	
1	0	3	5	8	9
2	4	4	6	8	
3	1	4	5		
4	5	7			

Key 0|6 means 6

(i) Determine the median score.

(2 marks)

(ii) Calculate the interquartile range of the scores.

(3 marks)

(iii) In the space below, construct a box-and-whisker plot to illustrate the data and comment on the shape of the distribution.

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(4 marks)

(b) Insecticides A, B or C are applied on lots Q, R and S. The same crop is planted on each lot and the lots are of the same size. The probability that a group of farmers will select A, B or C is 40%, 25% and 35% respectively. The probability that insecticide A is successful is 0.8, that B is successful is 0.65, and that C is successful is 0.95.

(i) Illustrate this information on a tree diagram showing ALL the probabilities on ALL branches.

(3 marks)

(ii) An insecticide is selected at random, determine the probability that it is unsuccessful.

(3 marks)

GO ON TO THE NEXT PAGE

(c) A regular six-sided die is tossed 2 times.

(i) Calculate the probability of obtaining a 5 on the 2nd toss, given that a 5 was obtained on the 1st toss.

(2 marks)

(ii) Determine the probability that a 5 is obtained on both tosses.

(2 marks)

(iii) Explain why the answers in (c) (i) and (c) (ii) are different.

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(1 mark)

Total 20 marks

8. (a) A particle moves in a straight line so that its distance, s metres, after t seconds, measured from a fixed point, O, is given by the function $s = t^3 - 2t^2 + t - 1$.

Determine

(i) its velocity when $t = 2$

(2 marks)

(ii) the values of t when the particle is at rest

(4 marks)

(iii) the distance between the rest points

(3 marks)

(iv) the time at which the maximum velocity occurs.

(3 marks)

(b) A bus starts from rest at Station A and travels a distance of 80 km in 60 minutes to Station B. Since the bus arrived at Station B early, it remained there for 20 minutes then started the journey to Station C. The time taken to travel from Station B to Station C was 90 minutes at an average speed of 80 kmh^{-1}.

(i) On the grid provided **on page 27**, draw a distance–time graph to illustrate the motion of the bus. **(3 marks)**

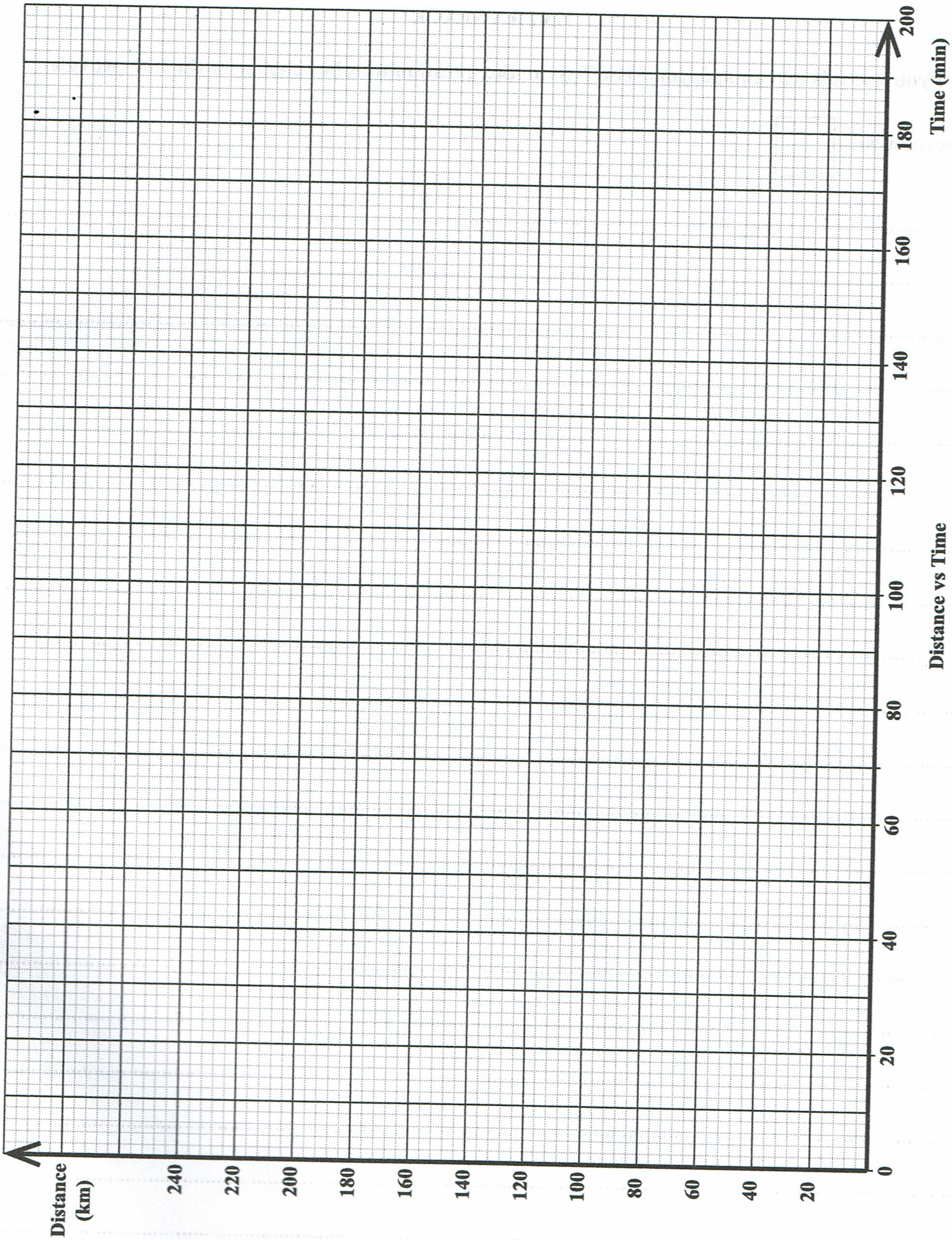
(ii) Determine the distance from Station B to Station C.

(2 marks)

(iii) Determine the average speed from Station A to Station B, in kmh^{-1}.

(3 marks)

Total 20 marks



END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.