TEST CODE **01254020**

FORM TP 2016037

MAY/JUNE 2016

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of FOUR sections. Answer ALL questions in Section I, Section II and Section III.
2. Answer ONE question in Section IV.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. A list of formulae is provided on page 4 of this booklet.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

<u>**Required Examination Materials**</u>

Electronic calculator (non programmable)
Geometry set
Mathematical tables (provided)
Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

LIST OF FORMULAE

Arithmetic Series $T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric Series $T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r},\ -1 < r < 1$ or $|r| < 1$

Circle $x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x+f)^2 + (y+g)^2 = r^2$

Vectors $\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)}$ where $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Differentiation

$$\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

Statistics

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\overline{x})^2$$

Probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Kinematics $v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2$

GO ON TO THE NEXT PAGE

SECTION I

Answer BOTH questions.

ALL working must be clearly shown.

1. (a) The domain for the function $f(x) = 2x - 5$ is $\{-2, -1, 0, 1\}$.

(i) Determine the range of the function.

....

....

....

....

....

(2 marks)

(ii) Find $f^{-1}(x)$.

....

....

....

(1 mark)

GO ON TO THE NEXT PAGE

(iii) Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same axes.

(2 marks)

(iv) Comment on the relationship between the two graphs.

....

(1 mark)

(b) Solve the equation $2^{2x+1} + 5(2^x) - 3 = 0$.

....

....

....

....

....

....

....

....

....

(4 marks)

DO NOT WRITE IN THIS AREA

(c) (i) Given that $T = k\,p^{\left(\frac{h}{c}\right)}$, make c the subject of the formula.

....

....

....

....

....

....

....

(2 marks)

(ii) Solve the equation

$$\log(x + 1) + \log(x - 1) = 2\log(x + 2).$$

....

....

....

....

....

....

....

....

(2 marks)

Total 14 marks

2. (a) (i) Determine the nature of the roots of the quadratic equation $2x^2 + 3x - 9 = 0$.

....

....

....

(1 mark)

(ii) Given that $f(x) = 2x^2 + 3x - 9$, sketch the graph of the quadratic function, clearly indicating the minimum value.

(5 marks)

(b) Evaluate $\sum_{1}^{25} 3^{-n}$.

....

....

....

....

....

(3 marks)

(c) A man invested \$$x$ in a company in January 2010, on which he earns quarterly dividends. At the end of the second, third and fourth quarter in 2011, he earned \$100, \$115 and \$130 respectively. Calculate the total dividends on his investment by the end of 2016.

....

....

....

....

....

....

....

....

....

(5 marks)

Total 14 marks

SECTION II

Answer BOTH questions.

ALL working must be clearly shown.

3. (a) (i) The points $M(3, 2)$ and $N(-1, 4)$ are the ends of a diameter of circle C. Determine the equation of circle C.

....

....

....

....

....

....

....

....

(5 marks)

(ii) Find the equation of the tangent to the circle C at the point $P(-1, 6)$.

....

....

....

....

....

....

....

(3 marks)

(b) The position vector of two points A and B, relative to a fixed origin, O, are $\bar{a}$ and $\bar{b}$ respectively, where $\bar{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, and $\bar{b} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$. P lies on $\overrightarrow{AB}$ such that $\overrightarrow{PB} = \frac{1}{4}\overrightarrow{AB}$.

Find the coordinates of $\overrightarrow{OP}$.

....

....

....

....

....

....

....

....

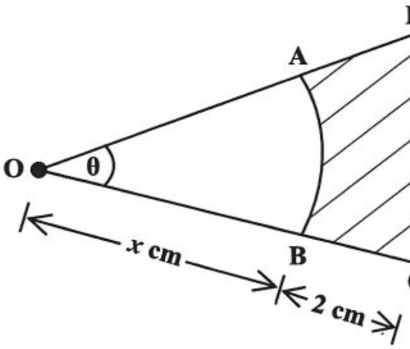
....

....

(4 marks)

Total 12 marks

4. (a) The following diagram (not drawn to scale) shows two sectors, AOB and DOC. OB and OC are x cm and $(x + 2)$ cm respectively and angle $AOB = \theta$.



If $\theta = \dfrac{2\pi}{9}$ radians, calculate the area of the shaded region in terms of x.

....

....

....

....

....

....

....

....

(4 marks)

(b) Given that $\cos 30° = \dfrac{\sqrt{3}}{2}$ and $\sin 45° = \dfrac{\sqrt{2}}{2}$, without the use of a calculator, evaluate $\cos 105°$, in surd form, giving your answer in the simplest terms.

....

....

....

....

....

....

(5 marks)

GO ON TO THE NEXT PAGE

(c) Prove that the identity $\frac{\sin(\theta + \alpha)}{\cos\theta \cos\alpha} \equiv \tan\theta + \tan\alpha$.

....

....

....

....

....

....

....

....

....

(3 marks)

Total 12 marks

SECTION III

Answer BOTH questions.

ALL working must be clearly shown.

5. (a) Find $\frac{dy}{dx}$ given that $y = \sqrt{5x^2 - 4}$, simplifying your answer.

....

....

....

....

....

....

(4 marks)

(b) The point P (1, 8) lies on the curve with equation $y = 2x(x + 1)^2$. Determine the equation of the normal to the curve at the point P.

....

....

....

....

....

....

....

....

....

....

(5 marks)

(c) Obtain the equation for EACH of the two tangents drawn to the curve $y = x^2$ at the points where $y = 16$.

....

....

....

....

....

....

....

....

....

....

....

....

(5 marks)

Total 14 marks

6. (a) (i) Find $\int (3\cos\theta - 5\sin\theta)\, d\theta$.

....

....

....

....

....

....

....

(3 marks)

(ii) Evaluate $\int_1^3 \left[\frac{2}{x^2} - 3 + 2x^3\right] dx$.

....

....

....

....

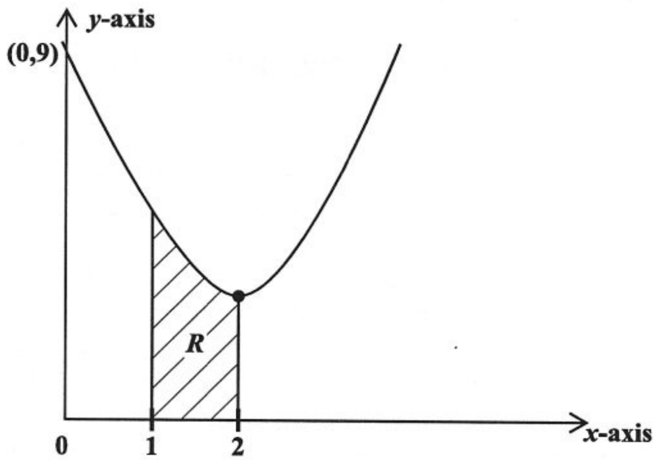
....

....

....

(4 marks)

(b) The following figure shows the finite region R bounded by the lines $x = 1$, $x = 2$ and the arc of the curve $y = (x - 2)^2 + 5$.



Calculate the area of the region R.

....

....

....

....

....

....

....

....

(4 marks)

(c) The point P (1, 2) lies on the curve which has a gradient function given by $\frac{dy}{dx} = 3x^2 - 6x$. Find the equation of the curve.

....

....

....

....

....

....

....

....

(3 marks)

Total 14 marks

SECTION IV

Answer only ONE question.

ALL working must be clearly shown.

7. (a) Use the data set provided below to answer the questions which follow.

15	16	18	18	20	21	22	22
22	25	28	30	30	32	35	40
41	52	54	59	60	65	68	75

(i) Construct a stem-and-leaf diagram to represent the given data.

(3 marks)

(ii) State an advantage of using the stem-and-leaf diagram to represent the given data.

....

....

(1 mark)

(iii) Determine the mode.

....

(1 mark)

(iv) Determine the median.

....

....

....

(2 marks)

(v) Determine the interquartile range.

....

....

....

....

....

(3 marks)

(b) Two events, A and B, are such that $P(A) = 0.5$, $P(B) = 0.8$ and $P(A \cup B) = 0.9$.

(i) Determine $P(A \cap B)$.

....

....

....

(2 marks)

(ii) Determine $P(A \mid B)$.

....

....

....

....

....

....

(2 marks)

(iii) State, giving a reason, whether or not A and B are independent events.

....

....

....

....

....

....

....

(2 marks)

(c) A bag contains 3 red balls, 4 black balls and 3 yellow balls. Three balls are drawn at random **with replacement** from the bag. Find the probability that the balls drawn are all of the same colour.

....

....

....

....

....

....

....

(4 marks)

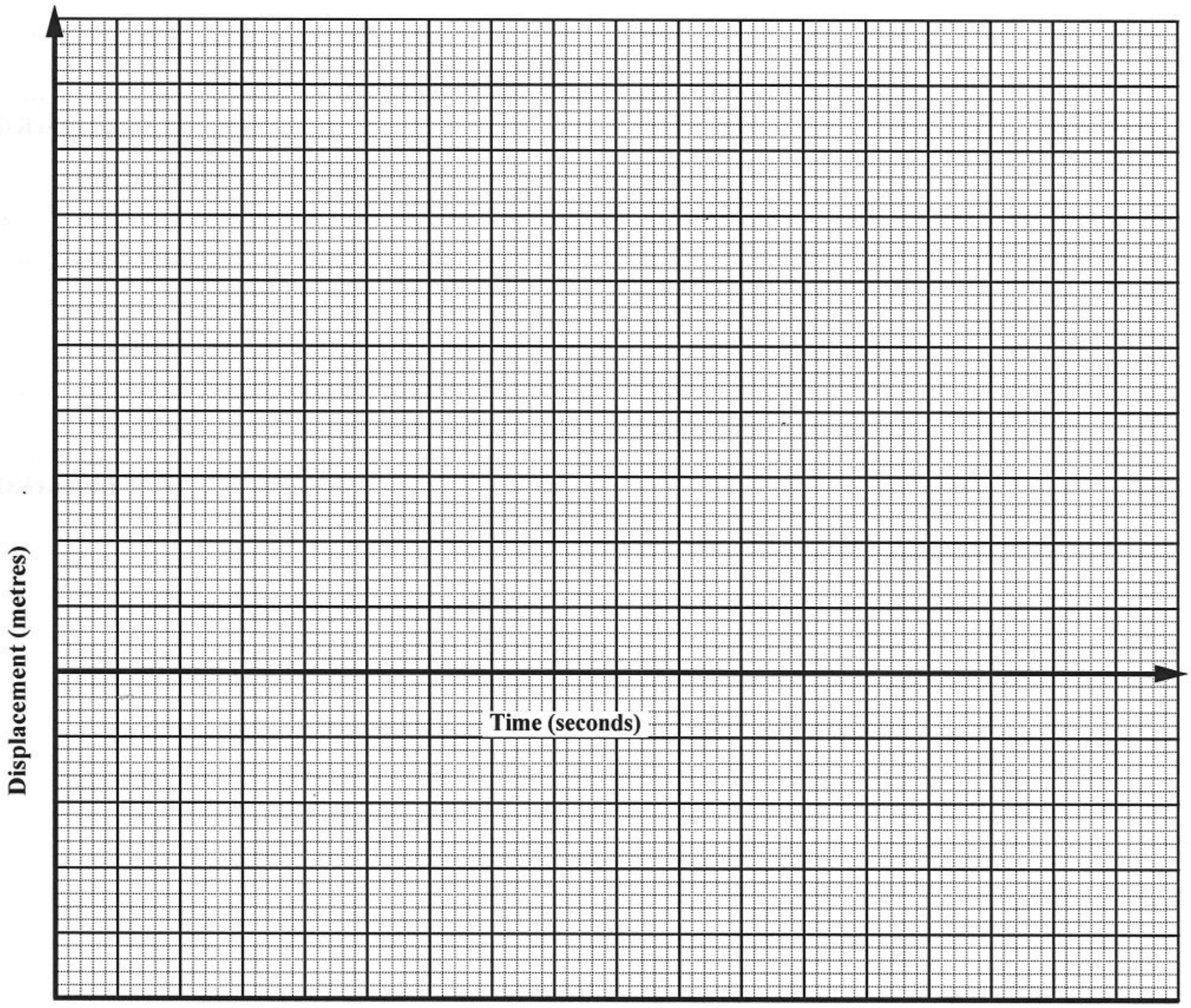
Total 20 marks

GO ON TO THE NEXT PAGE

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

8. (a) A motorist starts from a point, X, and travels 100 m due North to a point, Y, at a constant speed of 5 m s^{-1}. He stays at Y for 40 seconds and then travels at a constant speed of 10 m s^{-1} for 200 m due South to a point, Z.

(i) On the following grid, draw a displacement–time graph to display this information.



(5 marks)

(ii) Calculate the average speed for the whole journey.

....

....

....

....

(3 marks)

(iii) Calculate the average velocity of the whole journey.

....

....

....

....

(3 marks)

(b) A particle starting from rest, travels in a straight line with an acceleration, a, given by $a = \cos t$ where t is the time in seconds.

(i) Find the velocity of the particle in terms of t.

....

....

....

....

(3 marks)

(ii) Calculate the displacement of the particle in the interval of time $t = \pi$ to $t = 2\pi$.

....

....

....

....

....

....

....

....

(6 marks)

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.