

HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2019
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT I – PREVIEW TEST 2
1 hour 20 minutes

This examination paper consists of 2 printed pages.

This paper consists of 6 questions.

The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

- (i) Write your name clearly on each sheet of paper used
- (ii) Answer **ALL** questions
- (iii) Number your questions identically as they appear on the question paper and do **NOT** **write your solutions to different questions** beside each other
- (iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

- (a) Mathematical formulae
- (b) Scientific calculator (non-programmable, non-graphical)

1) Prove that $\frac{\cos A - \sin A}{1 - \tan A} \equiv \cos A$ [5]

2) (i) Prove that $\frac{\sin A + \sin 5A}{2\cos 2A} \equiv \sin 3A$. [4]

(ii) Hence, find the general solution of the equation $\frac{\sin A + \sin 5A}{2\cos 2A} = 1$. **Ans.** $\frac{n\pi}{3} + (-1)^n \frac{\pi}{6}$ [4]

3) (i) Express $3\sin x - \cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$, giving

the exact value of R , and the value of α correct to 3 decimal places. **Ans.** $\sqrt{10}, 0.322^\circ$ [6]

(ii) Hence, solve the equation $3\sin x - \cos x = \frac{\sqrt{3}}{2}$ for $0 \leq x \leq 2\pi$, giving your

answers correct to 3 decimal places. **Ans.** $0.599^\circ, 3.187^\circ$ [6]

4) Find, in exact form, the coordinates of the points of intersection of the two circles

$$x^2 + y^2 + 8x + 2y - 22 = 0 \text{ and } x^2 + y^2 - 6x + 4y + 2 = 0. \quad [7]$$

$$\text{Ans. } \left(\frac{73 \pm \sqrt{429}}{50}, \frac{-89 \pm 7\sqrt{429}}{50} \right)$$

5) (a) (i) A curve, C , is defined parametrically by the equations

$$x = 7\sin \theta - 5 \text{ and } y = 7\cos \theta + 2. \text{ Ans. } (x + 5)^2 + (y - 2)^2 = 49 \quad [4]$$

(ii) Describe fully, the locus of C . **Ans.** circle, centre $(-5, 2)$, radius 7 [3]

(b) A line, l , passes through the point $P(-5, 8, -3)$ and is parallel to the vector $-4\mathbf{i} - 7\mathbf{j} + 3\mathbf{k}$.

Find, for the line l

(i) its vector equation **Ans.** $\mathbf{r} = -5\mathbf{i} + 8\mathbf{j} - 3\mathbf{k} + \lambda(-4\mathbf{i} - 7\mathbf{j} + 3\mathbf{k})$ [2]

(ii) its cartesian equations **Ans.** $\frac{x+5}{-4} = \frac{y-8}{-7} = \frac{z+3}{3}$ [3]

(c) Find the angle, correct to 1 decimal place, between the lines with equations

$$\mathbf{r} = 2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k} + \lambda(-3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) \text{ and } \mathbf{r} = -4\mathbf{i} + 6\mathbf{j} - \mathbf{k} + \mu(5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) \text{ Ans. } 37.1^\circ [5]$$

6) A plane contains three non-collinear points $A(-4, -8, 3)$, $B(2, -7, 4)$ and $C(-3, 5, -1)$

(i) Prove that the vector $-17\mathbf{i} + 25\mathbf{j} + 77\mathbf{k}$ is normal to the plane [5]

(ii) Hence, obtain the Cartesian equation of the plane. **Ans.** $-17x + 25y + 77z = 99$ [3]

(iii) Find the perpendicular distance of the plane from the origin. **Ans.** $\frac{99}{\sqrt{6843}}$ [3]

END OF TEST