**HARRISON COLLEGE INTERNAL EXAMINATION APRIL 2022**

**CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**

**SCHOOL BASED ASSESSMENT**
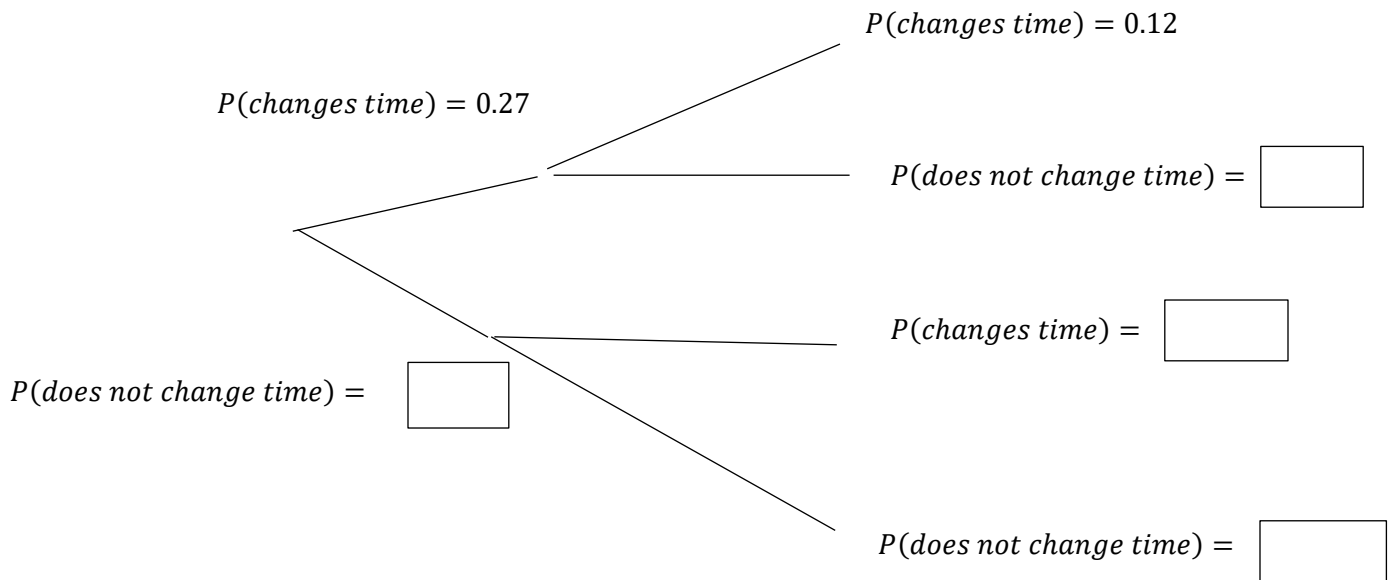
**PURE MATHEMATICS**

**UNIT II – TEST 3 <span style="color:red">PREVIEW</span>**

**Time: 1 hour and 20 minutes**

**1)** Dionne has a Zoom Maths lesson once a week with Ms. Layne. The probability that Ms. Layne changes the time of the meeting for the next week is $0.27$. If she changes the time for the meeting in the next week, the probability that she changes the time for the meeting in the following week is $0.12$. If she does not change the time for the meeting next week, the probability that she does not change the time for the meeting the following week is $0.65$.

a) Complete the probability tree diagram below to show all the possibilities for the next two weeks.     [4]

$P(changes\ time) = 0.12$

$P(changes\ time) = 0.27$

$P(does\ not\ change\ time) =$ ☐

$P(changes\ time) =$ ☐

$P(does\ not\ change\ time) =$ ☐

$P(does\ not\ change\ time) =$ ☐

b) Find the probability that Ms. Layne

    i) does not change the time for the next two meetings                                  [1]

    ii) changes the time for the second meeting                                       [2]

c) It is determined that the probability that Ms. Layne changes the time three weeks in a row is 0.0162. Calculate the probability that she changes the time in the first two weeks but does not change the time in the third week.                    [2]

**2)** a) Find the number of ways that the letters in **PREVIEW TEST** may be rearranged if

    i)       no restrictions apply                                              [2]

    ii)       there must be an $T$ at each end of the arrangement                [2]

    iii)      the three $Es$ must be together.                                   [2]

b) Find the probability that if the letters of **PREVIEW TEST** are rearranged randomly, the consonants will be together. (Give your answer as an exact fraction in its lowest terms)          [4]

**3)** The matrix $A$ is such that $A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 3 & 1 & 2 \end{pmatrix}$

i)     Show that the determinant of $A$ is 10.                                    [3]

ii)     It is known that the matrix of cofactors of $A$ is $\begin{pmatrix} 5 & -5 & -5 \\ 3 & 1 & -5 \\ -1 & 3 & 5 \end{pmatrix}$. Find $A^{-1}$.     [2]

iii)     Hence, solve the system of equations
$\begin{aligned} 2x - y + z &= 1 \\ x + 2y - z &= 1 \\ 3x + y + 2z &= -2 \end{aligned}$     [5]

**4)** A virus hits a local school campus. The medical authorities discover that students are either sick, well

or carriers of the virus. Data was collected on the proportion of juniors and seniors in each category.

The information is shown in the table below.

| Category | Junior | Senior |
|----------|--------|--------|
| Well     | 0.1    | 0.2    |
| Sick     | 0.3    | 0.45   |
| Carrier  | 0.6    | 0.35   |

The student population is distributed as follows:

|          | Junior | Senior |
|----------|--------|--------|
| **Males**   | 104 | 107 |
| **Females** | 80  | 103 |

a) Write the information above as two matrices **A** *and* **B** and calculate the matrix product **AB** which gives information on the status of the males and females on the campus. [6]

b) Determine, giving your answers to the nearest whole number,

i) The number of male carriers. [1]

ii) The number of sick females. [1]

iii) The number of well females. [1]

**5)** Show that the general solution of the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = \cos^3 x$$

$$is\ y = \frac{1}{2}\sin 2x + A\cos x, \quad where\ A\ is\ an\ arbitrary\ constant$$
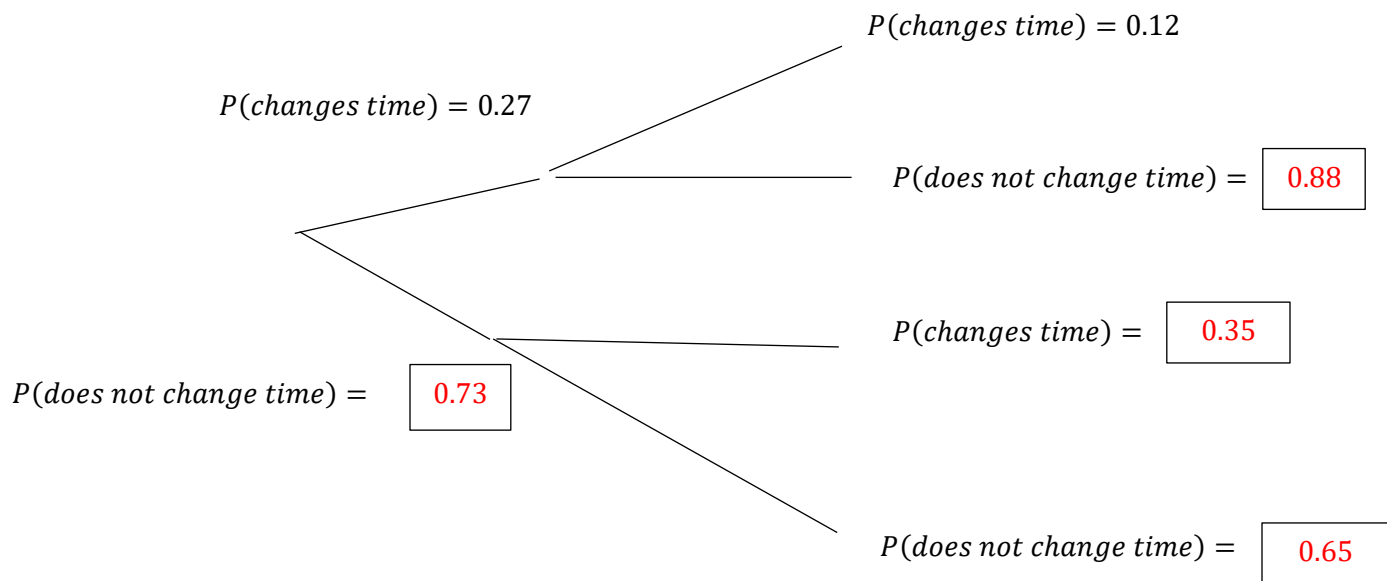
[8]

**6)** i) Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 3\cos x.$ [8]

ii) Find the particular solution that satisfies $y = 0\ and\ \frac{dy}{dx} = 0\ when\ x = 0.$ [6]

**End of Preview Test**

4

Answers

1 a)

$P(changes\ time) = 0.27$

$P(changes\ time) = 0.12$

$P(does\ not\ change\ time) = \boxed{0.88}$

$P(changes\ time) = \boxed{0.35}$

$P(does\ not\ change\ time) = \boxed{0.73}$

$P(does\ not\ change\ time) = \boxed{0.65}$

b)i) 0.475          ii) 0.288          c) 0.016

2) a) i) 3326400     ii) 60480     iii) 181440          b) $\frac{1}{66}$

3)ii) $\frac{1}{10}\begin{pmatrix} 5 & 3 & -1 \\ -5 & 1 & 3 \\ -5 & -5 & 5 \end{pmatrix}$          iii) $x = 1, y = -1, z = -2$

4   a) $\begin{pmatrix} 31.8 & 28.6 \\ 79.35 & 70.35 \\ 99.85 & 84.05 \end{pmatrix}$          b)i) 100        ii) 70          iii) 29

5) Proof

6) i) $y = Ae^{3x} + Be^{-x} - \frac{3}{10}\sin x - \frac{3}{5}\cos x$

ii) ) $y = \frac{9}{40}e^{3x} + \frac{3}{8}e^{-x} - \frac{3}{10}\sin x - \frac{3}{5}\cos x$