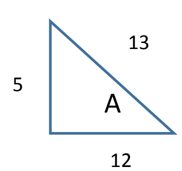
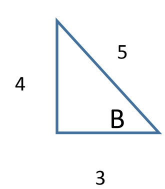
HARRISON COLLEGE INTERNAL EXAMINATION

CAPE PURE MATHEMATICS UNIT 1- TEST 2

1.	$3tan^2\theta + 4 = 1$ $3(sec^2\theta - 1) + 4sec\theta - 1 = 0$ $3sec^2\theta + 4sec\theta - 3 - 1 = 0$ $3sec^2\theta + 4sec\theta - 4 = 0$ $(3sec\theta - 2)(sec\theta + 2) = 0$ $sec\theta = \frac{2}{3}$ so $cos\theta = \frac{3}{2}$ Not possible OR $sec\theta = -2 \ so \ cos\theta = \frac{-1}{2}$ cos is negative so θ lies in the 2nd or 3rd quadrants $cos^{-1}\frac{1}{2} = \frac{\pi}{3}$ $\theta = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, \left(\pi + \frac{\pi}{3}\right) - 2\pi$ $= \frac{2\pi}{3}, \cancel{\frac{4\pi}{3}}, \frac{-2\pi}{3}$ (N.B. $\theta = \frac{4\pi}{3}$ $out \ of \ range$)	1 for identity $(tan^2\theta = sec^2\theta - 1)$ 1 for quadratic expression 1 1 1 1 for correct answer 1 for correct answer	7
2.	$sec2A + tan2A \equiv \frac{cosA + sinA}{cosA - sinA}$ LHS = $sec2A + tan2A$ $= \frac{1}{cos2A} + \frac{sin2A}{cos2A}$ $= \frac{1 + sin2A}{cos2A}$ $= \frac{1 + 2sinAcosA}{cos^2A - sin^2A}$ $= \frac{cos^2A + sin^2A + 2sinAcosA}{(cosA + sinA)(cosA - sinA)}$ $= \frac{\cancel{(cosA + sinA)}(cosA + sinA)}{\cancel{(cosA + sinA)}(cosA - sinA)}$ $= RHS$	1 A maximum of 5 marks for relevant steps to complete the proof 1 1 1 1	5
3.	$cosecA = \frac{13}{5}$ $sinA = \frac{5}{13}$ $cosB = \frac{3}{5}$ $cos(A + B) = cosAcosB - sinAsinB$ $= \frac{12}{13} \times \frac{3}{5} - \frac{5}{13} \times \frac{4}{5}$ $= \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$  	1 for value of $sinA$ 1 for 3rd side 1 for 3rd side 1 for value of $cosA$ 1 for formula 1 for final answer	6

4.	$5cosx - 3sinx \equiv Rcos(x+\alpha)$ $\equiv R[cosxcos\alpha - sinxsin\alpha]$		
	$5 = Rcos\alpha$ $3 = Rsin\alpha$ $\frac{3}{5} = \frac{\cancel{R}sin\alpha}{\cancel{R}cos\alpha}$ $= tan\alpha$ $\alpha = tan^{-1}\frac{3}{5}$ $= 31.0^0$ $25 = R^2cos^2\alpha$ $9 = R^2sin^2\alpha$ $34 = R^2(cos^2\alpha + sin^2\alpha)$ $R = \sqrt{34}$	1 for value of α 1 for value of R	
	$5cosx - 3sinx = \sqrt{34}cos(x + 31.0^0)$ $\sqrt{34}cos(x+31.0^0) = 4$ $cos(x+31.0^0) = \frac{4}{\sqrt{34}}$ $x + 31.0^0 = cos^{-1}\left(\frac{4}{\sqrt{34}}\right)$ $= 46.7^0, 360^0 - 46.7^0$ $= 46.7^0, 313.3^0$ $x = 46.7^0 - 31.0^0, 313.3^0 - 31.0^0$ $= 15.7^0, 282.3^0$	1 1 1 for correct value 1 for correct value	6
5.i)	$C_1: x^2 + y^2 - 6x - 4y + 9 = 0$ $x^2 - 6x + y^2 - 4y + 9 = 0$ $(x-3)^2 - 9 + (y-2)^2 - 4 + 9 = 0$ $(x-3)^2 + (y-2)^2 = 2^2$ Centre (3, 2) and radius 2 $C_2: x^2 + y^2 - 2x - 6y + 9 = 0$ $x^2 - 2x + y^2 - 6y + 9 = 0$ $(x-1)^2 - 1 + (y-3)^2 - 9 + 9 = 0$ $(x-1)^2 + (y-3)^2 = 1^2$ Centre (1, 3) and radius 1	1 for centre 1 for radius 1 for centre 1 for radius Marks also awarded if student showed that when *x=0* there was only 1 solution and when *y=0* there was also only 1 solution	4
ii)	$x^2 + y^2 - 6x - 4y + 9 = 0$ $x^2 + y^2 - 2x - 6y + 9 = 0$ Subtract $-4x + 2y = 0$ $2y = 4x$ $y = 2x$ $x^2 + (2x)^2 - 6x - 4(2x) + 9 = 0$ $x^2 + 4x^2 - 6x - 8x + 9 = 0$ $5x^2 - 14 + 9 = 0$ $(5x-9)(x-1) = 0$ $x = \frac{9}{5}$ $or\ x = 1$ $when\ x = \frac{9}{5}, y = \frac{18}{5}\ and\ when\ x = 1, y = 2$ Points of intersection $\left(\frac{9}{5}, \frac{18}{5}\right)\ and\ (1,2)$	1 for $x\ or\ y\ the\ subject\ of\ eqn$ 1 for substitution 1 for x value 1 for x value 1 for y value 1 for y value	6

iii)	Gradient of line $= \frac{\frac{18}{5}-2}{\frac{9}{5}-1} = \frac{8}{5} \div \frac{4}{5}$ $= \frac{8}{5} \times \frac{5}{4} = 2$ Equation of line looks like $y = 2x + c$ Substitute $(1,2)$ $to\ find\ c.\ 2 = 2(1) + c$ $c = 0\ so\ equation\ of\ line\ is\ y = 2x$	1 for gradient 1 for substitution 1 for equation	3
6 i)	Vector in direction of line $= \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ Vector equation of line $\boldsymbol{r} = \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ OR $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	1 for any multiple of vector 1 for position vector of point on line 1 for actual vector equation	3
ii)	$l_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2+\lambda \\ 4-\lambda \\ 4+\lambda \end{pmatrix}\ and\ l_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1+2\mu \\ 2-\mu \\ -3+4\mu \end{pmatrix}$ $2 + \lambda = 1 + 2\mu$ $4 - \lambda = 2 - \mu$ Solving simultaneously $\mu = 3\ and\ \lambda = 5$ Test for consistency using the 3rd equation $4 + \lambda = -3 + 4\mu$ $4 + 5 = -3 + 4(3)$ $9 = 9, consistent\ so\ lines\ intersect$	1 for value of λ 1 for value of μ 1 for check	3
iii)	$\boldsymbol{u} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}\ and\ \boldsymbol{v} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ $\boldsymbol{u.v} = \boldsymbol{u.v}$ $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = \sqrt{3}\sqrt{21} cos\theta$ $2 + 1 + 4 = \sqrt{63} cos\theta$ $\frac{7}{\sqrt{63}} = cos\theta , \theta = cos^{-1}\frac{7}{\sqrt{63}}$ $\theta = 28.1^0$	1 1 1	3
7 i)	$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ $= -\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + \begin{pmatrix} -2 \\ 26 \\ 11 \end{pmatrix} = \begin{pmatrix} -6 \\ 24 \\ 12 \end{pmatrix}$ Distance between A and $B = \lvert\overrightarrow{AB}\rvert$ $= \sqrt{36 + 576 + 144}$ $\sqrt{756} = 27.5\ m$	1 for formula 1 1	3

ii)	$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$ $-\begin{pmatrix}4\\2\\-1\end{pmatrix}+\begin{pmatrix}16\\17\\2\end{pmatrix}=\begin{pmatrix}12\\15\\3\end{pmatrix}$ $\begin{pmatrix}-6\\24\\12\end{pmatrix}.\begin{pmatrix}2\\-3\\7\end{pmatrix}=-12-72+84=0$ $\begin{pmatrix}12\\15\\3\end{pmatrix}.\begin{pmatrix}2\\-3\\7\end{pmatrix}=24-45+21=0$ Vectors are perpendicular since the dot product is 0 $\boldsymbol{r}.\begin{pmatrix}2\\-3\\7\end{pmatrix}=d$ $\begin{pmatrix}4\\2\\-1\end{pmatrix}.\begin{pmatrix}2\\-3\\7\end{pmatrix}=8-6-7=-5$ Equation of plane is $\boldsymbol{r}.\begin{pmatrix}2\\-3\\7\end{pmatrix}=-5$	1 1 1 1 for using any point on plane 1	5
iii)	$\overrightarrow{DE} = \overrightarrow{DO} + \overrightarrow{OE} = -\begin{pmatrix}-130\\40\\-20\end{pmatrix}+\begin{pmatrix}-90\\20\\-15\end{pmatrix}=\begin{pmatrix}40\\-20\\5\end{pmatrix}$	1	1
iv)	Vector normal to plane = $\begin{pmatrix}2\\-3\\7\end{pmatrix}$ $\begin{pmatrix}40\\-20\\5\end{pmatrix}.\begin{pmatrix}2\\-3\\7\end{pmatrix}=\sqrt{2025}\sqrt{62}\,cos\theta$ $\frac{80+60+35}{\sqrt{2025\times 62}}=cos\theta$ $\theta = cos^{-1}\left(\frac{175}{\sqrt{125550}}\right)=60.4^0$ Acute angle between cable and slope $=90^0-60.4^0=29.6^0$	1 for recognising normal 1 for formula 1 1 1	5