CAPE PURE MATHEMATICS
UNIT II – PREVIEW TEST 3
1 hour 20 minutes

NAME OF STUDENT: ____

SCHOOL CODE: 030014

DATE: ______________________

This examination paper consists of 8 printed pages and 2 blank pages for extra working.
This paper consists of **6** questions.
The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly in the space provided above.

2. Answer **ALL** questions in the **SPACES PROVIDED**.

If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided. You must also write your name and candidate number clearly on any additional paper used.

3. Number your questions carefully and **DO NOT write your solutions to different questions beside each other**.

4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

(a) Mathematical formulae
(b) Scientific calculator (non-programmable, non-graphical)

1) (a) Find the total number of permutations of the letters of the word **TESTINGS**. Ans $\frac{8!}{2!2!}$ [2]

(b) Find the number of these permutations which

(i) start and finish with **T**. Ans $\frac{6!}{2!}$ [2]

(ii) have the letters **E**, **S**, **S** together. Ans $\frac{6!}{2!} \times \frac{3!}{2!}$

(iii) have no TWO of the letters **E**, **S**, **S**, **I** consecutive. Ans $\frac{4!}{2!} \times \frac{4!}{2!}$ [2]

2) A squash club has 13 members, 7 are men and 6 are women. A team of 6 members is selected to play in a tournament.

(a) Find the number of different ways of selecting the team if

(i) all the players are to be of the same sex. Ans $7_{C_6} + 6_{C_6}$ [2]

(ii) there must be an equal number of men and women. Ans $7_{C_3} \times 6_{C_3}$ [2]

(b) Given that the 7 men include 2 brothers, find the total number of ways in which the team can be selected if either of the brothers, but not both, must be included. Ans $2_{C_1} \times 11_{C_5}$ [3]

3) An agricultural officer measures the heights of seedlings in a research facility, and records the colour of the seedlings in the table below.

	Height, h, of tree in centimetres		
	$h < 2$	$2 < h < 4$	$h > 4$
Pale green	2	15	3
Off white	11	21	8

A dendrologist chooses a seedling at random for monitoring.

Determine the probability that the chosen seedling is

(a) off white. Ans. $\frac{40}{60}$ [1]

(b) Over 4 centimetres high. Ans. $\frac{11}{60}$ [1]

(c) pale green and less than 4 centimetres high. Ans. $\frac{17}{60}$ [1]

(d) Over 2 centimetres high, given that it is pale green. Ans. $\frac{18}{20}$ [2]

4) (a) Solve for x, $\begin{vmatrix} 2 & 1 & x-1 \\ x+2 & 3 & 4 \\ x+1 & 1 & 2 \end{vmatrix} = 0$ Ans. $-1, \frac{5}{2}$; [10]

(b) Given the system of equations
$$\begin{aligned} x - y - z &= 3 \\ 2x + y - z &= 10 \\ x - 2y + 3z &= 6 \end{aligned}$$

(i) Write the augmented matrix. [2]

(ii) Reduce the augmented matrix to echelon form. [3]

(iii) Hence, determine the solutions if any, of the system of equations. Ans. 5,1,1 [3]

5) (i) The cost $\$c$ of manufacturing x items may be modelled by the differential equation $\frac{dc}{dx} + 2c = 10x$. By using a suitable integrating factor, find the general solution of the differential equation. Ans. $ce^{2x} = 5(xe^{2x} - \frac{1}{2}e^{2x}) + k$ [6]

(ii) Given that there is a cost of \$100 when no items are produced, find the particular solution. Ans. $ce^{2x} = 5(xe^{2x} - \frac{1}{2}e^{2x} + \frac{41}{2})$ [2]

6) The general solution of the differential equation $y'' + 2y' + 5y = 4 \sin 2t$ is $y = CF + PI$, where CF is the complementary function and PI is a particular integral.

(i) a) Calculate the roots of $\lambda^2 + 2\lambda + 5 = 0$, the auxiliary equation. [3]

b) Hence, obtain the complementary function (CF) of $y'' + 2y' + 5y = 0$. [3]

(ii) Given that the form of the particular integral (PI) is $y(t) = A\cos 2t + B\sin 2t$, show that $A = -\frac{16}{17}$ and $B = \frac{4}{17}$. [5]

(iii) Given that $y(0) = 0.04$ and $y'(0) = 0$, obtain the particular solution of the differential equation. [5]

Ans. $-1 \pm 2i$, $y = e^{-t}[C\cos 2t + D\sin 2t]$,
$y(t) = e^{-t}[\frac{417}{425} cos2t + \frac{217}{850} sin2t] - \frac{16}{17} cos2t + \frac{4}{17} sin2t$

End of Test