FORM TP 2024324

TEST CODE 02134020

MAY/JUNE 2024

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

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READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s), you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

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Examination Materials Permitted

Mathematical formulae and tables (provided) – Revised 2012
Mathematical instruments
Silent, non-programmable electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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SECTION A

Module 1

Answer BOTH questions.

1. (a) Given that $f(x) = \frac{4}{x-5}$ and $g(x) = \frac{1}{x}$, determine the domain of the composite function $f(g(x))$.

[5 marks]

(b) Solve the inequality $-3|2x - 5| + 2 \geq -4$.

[4 marks]

(c) Solve the logarithmic equation $\log_5 x - 4 \log_x 5 = -3$.

[8 marks]

(d) Prove that the function $f(x) = 3x - 2$ is bijective.

[8 marks]

Total 25 marks

2. (a) The roots of the cubic equation $x^3 - 9 = 0$ are α, β and γ. Determine the cubic equation with roots $(\alpha - 1)$, $(\beta - 1)$ and $(\gamma - 1)$, given that
$(\alpha - 1)(\beta - 1) + (\alpha - 1)(\gamma - 1) + (\beta - 1)(\gamma - 1) = 12$.

[8 marks]

(b) The function $f(x) = 2x^3 - px^2 + qx - 5$ is divisible by $2x - 1$ and has a remainder of 7 when divided by $x - 1$. Calculate the values of p and q.

[9 marks]

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(c) Let P be the proposition that if $n^3 + 3$ is odd then n is even for all integers n. Provide a proof by contradiction for this proposition.

Hint: Let $n^3 + 3$ be odd and assume that n is odd. Show that the assumption that n is odd is incorrect.

[8 marks]

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) (i) Express $5 \sin \theta + 12 \cos \theta$ in the form $r \sin(\theta + \alpha)$, where $r > 0$ and $0 < \alpha < 2\pi$.

[6 marks]

(ii) Hence, or otherwise, show that $5 \sin \theta + 12 \cos \theta + 7 \leq 20$.

[3 marks]

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(b) (i) Show that $2 \sin x + 3 \cos x = 3$ may be written as $13 \cos^2 x - 18 \cos x + 5 = 0$.

[5 marks]

(ii) Hence or otherwise, solve the equation $2 \sin x + 3 \cos x = 3$ for $-\pi \leq x \leq \pi$.

[5 marks]

(c) Show that $\frac{(2\cos^2 x - 1)^2}{\cos^4 x - \sin^4 x} \equiv 1 - 2\sin^2 x$.

[6 marks]

Total 25 marks

4. (a) A plane, P, passes through the point $(5, 1, -3)$ and is perpendicular to the vector $n = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

(i) Show that the equation of the plane is $3x - 4y + 2z - 5 = 0$.

[4 marks]

(ii) Calculate the angle, in radians, between P and the plane with equation $7x - 4y + 3z = 5$.

[6 marks]

(b) The equation of a circle is given as $x^2 + y^2 - 6x - 8y - 39 = 0$.

(i) Determine the centre and radius of the circle.

[5 marks]

(ii) Determine the equation of the tangent to the circle at the point (11, 4).

[5 marks]

(c) A point, Q, moves in the x–y plane such that it is equidistant from $A(2,-5)$ and $B(-2, 3)$. Determine the locus of Q.

[5 marks]

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) A function f is defined as $f(x) = \begin{cases} (x+4)^3 & , x < -2 \\ -10 & , x = -2 \\ x^2 + 8x - 1 & , x > -2. \end{cases}$

(i) Determine $f(-2)$.

[1 mark]

(ii) Determine $\lim_{x \to -2} f(x)$.

[5 marks]

(iii) Hence, or otherwise, determine whether f is continuous at $x = -2$.

[1 mark]

(b) Evaluate $\lim_{\theta \to 0} \left(\frac{\sin \theta}{\sin 4\theta} \right)$.

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[4 marks]

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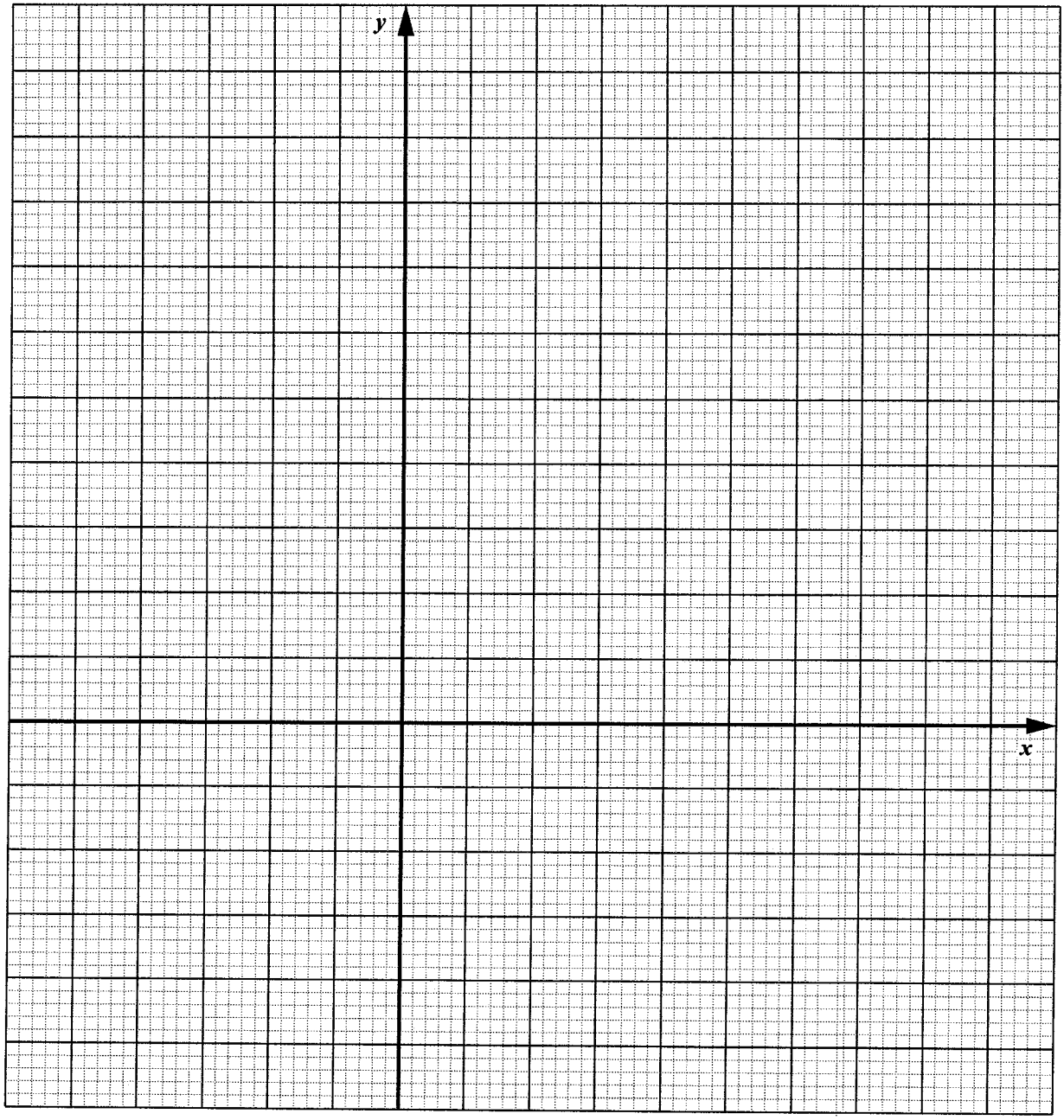
(c) A function, g, is defined as $g(x) = x^3 - 3x^2$.

(i) Determine the stationary points of g and the nature of the stationary points.

[9 marks]

(ii) Hence, or otherwise, sketch the graph of g showing its intercepts, stationary points and any other important features.



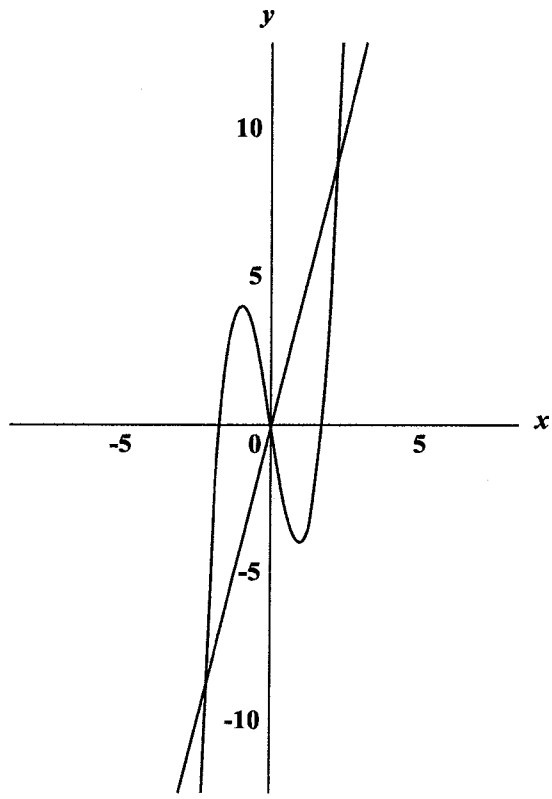
[5 marks]

Total 25 marks

6. (a) Calculate the volume of the solid generated by revolving the region bounded by the graphs of $x = \sqrt{y}$ and $y = 2x$ about the y-axis.

[7 marks]

(b) The diagram below, not drawn to scale, shows the graphs of $y = 2x^3 - 6x$ and $y = 4x$.



Calculate the area between the line and the curve.

[7 marks]

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(c) Given that f is an **even** function, show that $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

[5 marks]

(d) Solve the differential equation $x^3y' = 2 - x^4$, given that at $x = 1$, $y = 2$.

[6 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.